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The Daugavet property for pairs of Banach spaces

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We prove that if a separable Banach space Y contains $C[0,1]$, then Y can be renormed so that the pair $(C[0,1], Y)$ has the Daugavet property with respect to narrow operators. A similar result is proved for $L_1[0,1]$.

1. Introduction

This paper deals with a property of Banach spaces. Namely, we say that a pair (X, Y), where $X \subset Y$, has the Daugavet property with respect to some class of operators $\mathcal{M} \subset \mathcal{L}(X, Y)$ and write $(X, Y) \in DPr$ if any $T \in \mathcal{M}$ satisfies Daugavet equation

$$\|T + J\| = 1 + \|T\|,$$

where $J : X \mapsto Y$ is the natural embedding. Sometimes we use this concept when X is isometrically embedded into Y. In this case, a role of J plays given embedding.

A question considered here concerns a problem of renorming Y so that $(X, Y) \in DPr$ with respect to some class of operators. The result stated in abstract has some interesting consequences concerning isomorphic properties of $C[0,1]$ and $L_1[0,1]$ (C and L_1 for short).

In Sections 4 and 5 some propositions we need are formulated without proofs.* They are given in [1] and [3].

Our notation is standard.

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2. Preliminary constructions

First, we deal with two properties of second category sets in metric spaces. They will be useful in the sequel.

Let K be a metric space with a metric ρ. We shall use the following notation:

$$B(k,r) = \{k' \in K : \rho(k',k) < r\},$$

$$S(k,r) = \{k' \in K : \rho(k',k) = r\}.$$

Denote by $\overline{A}$ the closure of A and $\widehat{A} = K \setminus A$.

Let us recall that $A \subset K$ is said to be a first category (f.c.) set if $A = \cup_{i=1}^{\infty} A_i$, where A_i are nowhere-dense sets. In the converse case, A is called a second category (s.c.) set.

Lemma 1. *Let $B \subset K$ be a s.c. set; then there exists a point $k_0 \in K$ and a neighborhood $U(k_0)$ of k_0 such that for each $k \in U(k_0)$ and each neighborhood $U(k)$ of k the set $U(k) \cap B$ is of s.c.*

The proof is not difficult (see [2]).

Corollary 2. *Suppose K does not contain an isolated point; then each s.c. set $B \subset K$ can be partitioned into countably many pairwise disjoint s.c. sets.*

P r o o f. Actually, it is sufficient to show that B contains two disjoint s.c. sets. By Lemma 1 we may fix k_0 and $U(k_0)$. Since K does not contain an isolated point, we have at least two points k_1, k_2 in $U(k_0)$. Thus, the sets

$$B(k_1, \frac{1}{3}\rho(k_1,k_2)) \cap B \text{ and } B(k_2, \frac{1}{3}\rho(k_1,k_2)) \cap B$$

are desired ones. The proof is complete. ∎

Now for any separable Banach space X we construct a Banach space m_0 satisfying the following conditions:

1) X is isometrically embedded into m_0 by an operator Q;

2) given a separable Banach space Y containing X, there is an isomorphic embedding $U : Y \mapsto m_0$ such that $U|_X = Q$.

The operator U gives a possibility to introduce an equivalent norm on Y : $p(x) = \|Ux\|$. We shall show that for L_1 and C this norm has the property we need (Sections 4 and 5).

Denote by K the closed unit ball of X^*. Since X is separable, the weak* topology of K is metrizable. In the sequel, only weak* topology on K is considered, and $\rho(\cdot,\cdot)$ is a metric which generates the topology.

Let us introduce the following Banach spaces:

$$l_\infty(K) = \{f : K \mapsto R, \quad \sup\{|f(s)|, s \in K\} = \|f\|_\infty < \infty\},$$

$$m(K) = \{f \in l_\infty(K) : \text{supp}(f) \ - \text{f.c.set}\}.$$

Lemma 3. *$m(K)$ is a closed linear subspace of $l_\infty(K)$.*

P r o o f. This can be deduced from the following simple statements:
if $f \equiv 0$, then $f \in m(K)$;
$\operatorname{supp}(\lambda f) = \operatorname{supp}(f)$, $\lambda \neq 0$;
$\operatorname{supp}(f_1 + f_2) \subset \operatorname{supp}(f_1) \cup \operatorname{supp}(f_2)$;
if $\|f_n - f\| \to 0$, as $n \to \infty$, then $\operatorname{supp}(f) \subset \cup_{n=1}^{\infty} \operatorname{supp}(f_n)$. ■

So we can construct the Banach space $m_0(K) = l_\infty(K)/m(K)$ with the norm $\|[f]\| = \inf\{\sup\{|f(s)|, s \in K \backslash F\}, F \text{ is a f.c. set }\}$.

Lemma 4. *X is isometrically embedded into $m_0(K)$.*

P r o o f. Since X is a subspace $C(K)$, it is sufficient to show that $C(K)$ is isometrically embedded into $m_0(K)$ by the quotient map. For this purpose we prove that $\|f\|_\infty = \|[f]\|$, $f \in C(K)$. In fact, $\|[f]\| \leq \|f\|_\infty$ is obvious. To prove the converse inequality, we fix an $\varepsilon > 0$ and a point $s_0 \in K$ such that $\|f\|_\infty = |f(s_0)|$. Then there is $\delta > 0$ such that $|f(s)| > \|f\|_\infty - \varepsilon$ for all $s \in B(s_0, \delta)$. Since $B(s_0, \delta)$ is a s.c. set, we obtain $\|f\|_\infty - \varepsilon \leq \inf\{\sup\{|f(s)|, s \in B(s_0, \delta) \setminus F\}, F$ is a f.c. set$\} \leq \|[f]\|$. So the quotient map restricted to X organizes the required embedding. ■

We will consider below the inclusions $X \subset C(K) \subset m_0(K)$ in the sense described.

Thus, Lemma 4 proves condition 1). The following theorem shows that the condition 2) is also valid for $m_0(K)$.

3. The embedding theorem

Theorem 5. *Let X, Y be separable Banach spaces, $X \subset Y$, $K = \overline{B}(X^*)$ and $Q : X \mapsto m_0(K)$ be the quotient map restricted to X. Then there exists an isomorphic embedding $U : Y \mapsto m_0(K)$ such that $U|_X = Q$.*

The statement of this theorem follows immediately from

Lemma 6. *Suppose $I = \{y_n\}_{n=1}^{\infty}$ be a dense set in $S(Y)$; then there exist s.c. sets $\{B_n\}_{n=1}^{\infty}$ in K such that for each $s \in B_n$ there is an extension $\widetilde{\delta}_s \in Y^*$ of δ_s satisfying:*
(a) $\|\widetilde{\delta}_s\| \leq 5$,
(b) $|\widetilde{\delta}_s(y_n)| \geq \frac{1}{2}$.
Moreover, for all $s \in B_i \cap B_j$ the corresponding extensions of δ_s coincide.

Let us prove Theorem 5 assuming the truth of Lemma 6.

Given an $s \in K \setminus \cup_{n=1}^{\infty} B_n$, we extend the δ_s to $\widetilde{\delta}_s \in Y^*$ by Hahn–Banach theorem. The required embedding can be defined in the following way:

$$Uy = [f], \text{ where } f(s) = \widetilde{\delta}_s(y).$$

The linearity of U is obvious. The continuity follows from the condition (a) of Lemma 6. Since $\widetilde{\delta}_s(x) = \delta_s(x) = x(s)$ for all $x \in X$, we obtain $U|_X = Q$. To prove that U is bounded from below we show that $\|Uy_n\| \geq \frac{1}{2}$, $n \in N$. Indeed, by the condition (b)

$$\|Uy_n\| \geq \inf\{\sup\{|\widetilde{\delta}_s(y_n)|, s \in B_n \setminus F\}, F \ - \text{f.c.set}\} \geq \frac{1}{2}.$$

This concludes the proof of Theorem 5.

P r o o f o f L e m m a 6. Let us divide I into two sets

$$I_1 = \{y_n^1 \in I : \exists x_n \in X, \|x_n - y_n^1\| < \frac{1}{4}\},$$

$$I_2 = \{y_n^2 \in I : \text{dist}(y_n^2, X) \geq \frac{1}{4}\}.$$

Denote by $\widetilde{\widetilde{\delta}}_s$ an extension of δ_s to Y by Hahn–Banach theorem. We fix a ball $B = \overline{B}(s', r') \subset K$ with the following property: if $s \in B$, then $-s \notin B$.

First, we construct $\{B_n^1\}_{n=1}^{\infty}$ corresponded to I_1. For this, we consider a functional $s_n \in S(X^*) \backslash B$ such that $|s_n(x_n)| = |x_n(s_n)| = \|x_n\| > \frac{3}{4}$. Since $x_n \in C(K)$ and $s_n \in K \setminus B$, we can find $r_n > 0$ such that

$$\overline{B}(s_n, r_n) \cap B = \emptyset$$

and

$$|\delta_s(x_n)| = |x_n(s)| > \frac{3}{4}, \text{ for all } s \in \overline{B}(s_n, r_n) = B_n^1.$$

We put $\widetilde{\delta}_s = \widetilde{\widetilde{\delta}}_s$ for $s \in B_n^1$. Thus $\|\widetilde{\delta}_s\| = 1$ and (a) follows. Moreover, the estimate

$$|\widetilde{\delta}_s(y_n^1)| \geq |\delta_s(x_n)| - |\widetilde{\delta}_s(x_n - y_n^1)| \geq \frac{1}{2}$$

implies (b).

Now, we construct $\{B_n^2\}_{n=1}^{\infty}$ corresponded to I_2. By Corollary 2, we can divide the s.c. set B into s.c. sets $\{B_n^2\}_{n=1}^{\infty}$. Let us extend $\delta_s, s \in B_n^2$ to Y as follows:

$$\widetilde{\delta}_s = \widetilde{\widetilde{\delta}}_s + \phi, \text{where } \phi \in Y^* : \ \phi|_X \equiv 0, \ \|\phi\| \leq 4,$$

and

$$\phi(y_n^2) = \begin{cases} \operatorname{sign}\tilde{\tilde{\delta}}_s(y_n^2), & \tilde{\tilde{\delta}}_s(y_n^2) \neq 0, \\ 1, & \tilde{\tilde{\delta}}_s(y_n^2) = 0. \end{cases}$$

The existence of ϕ follows from Hahn–Banach theorem. It is easily seen that $\tilde{\tilde{\delta}}_s$ is the required extension.

Finally, if $s \in B_i^1 \cap B_j^1$, then the corresponding extensions coincide. The other intersections are empty. ∎

4. The renorming theorem for $C[0,1]$

We start with some definitions and notation. Let $[a,b] \subset [0,1]$; denote by $C_0[a,b] \subset C$ a subspace of functions vanishing outside of $[a,b]$.

Definition 1. *We say that $T \in \mathcal{L}(C,Y)$ is narrow if, for each $[a,b] \subset [0,1]$ and each $\varepsilon > 0$, there is an $f \in S(C_0[a,b])$ such that $\|Tf\| < \varepsilon$.*

Let us mention some properties of narrow operators:

(N4.1) The function f in definition 1 can be assumed nonnegative;

(N4.2) If $T \in \mathcal{L}(C,Y)$ does not fix a copy of C, then T is narrow;

(N4.3) Narrow operators form a closed linear subspace of $\mathcal{L}(C,Y)$;

(N4.4) The space of narrow operators is a left-hand ideal in the following sense: if $T \in \mathcal{L}(C,Y)$ is narrow and $U \in \mathcal{L}(Y,Z)$, then $UT \in \mathcal{L}(C,Z)$ is narrow.

These and other properties of narrow operators are considered in [1] in detail.

Definition 2. *A point $t \in [0,1]$ is called a vanishing point of $T \in \mathcal{L}(C,Y)$ ($t \in \operatorname{van}T$) if there are intervals $[\alpha_n, \beta_n] \subset [0,1]$ and nonnegative functions $f_n \in S(C_0[\alpha_n,\beta_n])$, $n = 1,2,\ldots$ satisfying the following conditions:*

- $\lim\limits_{n\to\infty} f_n(\tau) = \delta_{t\tau}$, *where $\delta_{t\tau}$ is the Kronecker delta;*
- $\alpha_n, \beta_n \to t$ $(n \to \infty)$*;*
- $\|Tf_n\| \to 0$ $(n \to \infty)$.

Using property (N4.1), the reader will easily prove that T is narrow iff $\overline{\operatorname{van}T} = [0,1]$.

We apply below the construction of the embedding Q of Lemma 4 for $X = C$. So, $K = \overline{B}\,(C[0,1]^*)$ is the set of Borel measures. The following theorem contains almost complete proof of the main result.

Theorem 7. *$(C, m_0(K)) \in DPr$ with respect to narrow operators, where $K = \overline{B}(C^*)$.*

P r o o f. Let $T \in \mathcal{L}(C, m_0(K))$ be a narrow operator. Given an $\varepsilon > 0$, there is an $f \in S(C)$ such that $\|Tf\| > \|T\| - \varepsilon$. Let us consider a set

$$B = \{\mu \in K : (Tf)(\mu) > \|Tf\| - \varepsilon\}.$$

Without loss of generality, it can be assumed that B is a s.c. set. Now, pick a $\mu_0 \in K$ and a weak*-neighborhood $U(\mu_0)$ using Lemma 1.

The only extreme points of $\overline{B}(C^*)$ are $\pm\delta$-measures $\delta_t \in C^* : \langle \delta_t, f\rangle \overset{\text{def}}{=} f(t)$. Then, by Krein–Milman theorem, there are

$$\lambda_1, \lambda_2, \ldots, \lambda_n \in R \setminus \{0\} \quad \text{and} \quad t_1, t_2, \ldots, t_n \in [0, 1]$$

such that

$$\sum_{k=1}^{n} |\lambda_k| = 1 \quad \text{and} \quad \mu' = \sum_{k=1}^{n} \lambda_k \delta_{t_k} \in U(\mu_0).$$

Since van$T$ is dense in $[0, 1]$, we can assume that $t_k \in \text{van}T$, $k = \overline{1, n}$. Then, by definition 2, there are disjoint intervals $I_k = [\alpha_k, \beta_k]$ and nonnegative functions

$$f_k \in (1 - f(t_k) \cdot \text{sign}\lambda_k) S(C_0(I_k)), \tag{1}$$

satisfying

$$f_k(t_k) > 1 - f(t_k) \cdot \text{sign}\lambda_k - \varepsilon \tag{2}$$

and

$$\|Tf_k\| < \frac{\varepsilon}{n} \tag{3}$$

for each t_k. Furthermore, we may assume that

$$|f(t) - f(t_k)| < \varepsilon, \ t \in I_k. \tag{4}$$

Now, we modify f putting

$$\widetilde{f} = f + \sum_{k=1}^{n} f_k \cdot \text{sign}\lambda_k.$$

Describe some properties of the $\widetilde{f}$. First,

$$\|\widetilde{f}\| \leq 1 + \varepsilon. \tag{5}$$

Indeed, by choice of f_k we have

$$|\widetilde{f}(t)| = |f(t)| \leq 1 \ for \ t \in [0, 1] \setminus \cup_{k=1}^{n} I_k$$

and if $t \in I_k$ for some k, then by (1) and (4) we obtain

$$|\widetilde{f}(t)| \leq |f(t_k) + f_k(t) \cdot \text{sign}\lambda_k| + |f(t) - f(t_k)| \leq |f(t_k) \cdot \text{sign}\lambda_k + f_k(t)| + \varepsilon \leq 1 + \varepsilon.$$

Secondly, it follows immediately from (3) that

$$\|Tf - T\widetilde{f}\| < \varepsilon. \tag{6}$$

Thirdly, regarding $\widetilde{f}$ as an element of $C(K)$, we obtain by (2) the following estimate:

$$\widetilde{f}(\mu') \geq 1 - 2\varepsilon. \tag{7}$$

Actually,

$$\widetilde{f}(\mu') = \mu'(\widetilde{f}) = \sum_{k=1}^{n} \lambda_k \delta_{t_k}(\widetilde{f}) = \sum_{k=1}^{n} \lambda_k \widetilde{f}(t_k)$$

$$\geq \sum_{k=1}^{n} \lambda_k (f(t_k) + f_k(t_k) \cdot \text{sign}\lambda_k) - \varepsilon$$

$$= \sum_{k=1}^{n} |\lambda_k| (f(t_k) \cdot \text{sign}\lambda_k + f_k(t_k))\varepsilon$$

$$\geq \sum_{k=1}^{n} |\lambda_k| (f(t_k) \cdot \text{sign}\lambda_k + 1 - f(t_k) \cdot \text{sign}\lambda_k - \varepsilon) - \varepsilon = 1 - 2\varepsilon.$$

An intersection of B with any weak*-neighborhood of μ' is a s.c. set by the choice of $U(\mu_0)$. Thus, the set

$$A = \left\{ \mu \in B : |\mu(\widetilde{f}) - \mu'(\widetilde{f})| < \varepsilon \text{ and } \|Q\widetilde{f} + Tf\| \geq |\widetilde{f}(\mu) + (Tf)(\mu)| \right\}$$

is nonempty. Let us fix $\mu \in A$; then taking into account (5),(6) and (7), we have

$$(1+\varepsilon)\|Q + T\| \geq \|Q\widetilde{f} + T\widetilde{f}\| \geq \|Q\widetilde{f} + Tf\| - \varepsilon \geq |\widetilde{f}(\mu) + (Tf)(\mu)| - \varepsilon$$

$$\geq |\widetilde{f}(\mu') + (Tf)(\mu)| - 2\varepsilon \geq 1 + \|Tf\| - 3\varepsilon \geq 1 + \|T\| - 4\varepsilon$$

for arbitrary $\varepsilon > 0$. This completes the proof. ■

Now we prove our main result.

Theorem 8. *Let Y be a separable Banach space containing C as a subspace and let $J : C \mapsto Y$ be the natural embedding. Then Y can be renormed so that the new norm coincides with the original one on C and $(C, Y) \in DPr$ with respect to narrow operators in the new norm.*

P r o o f. By Theorem 5, there exists an isomorphic embedding $U : Y \mapsto m_0(K)$, $K = \overline{B}(X^*)$ such that $Uf = Qf$ for $f \in C$. Put $|||y||| = \|Uy\|$, $y \in Y$. Then, given a narrow operator $T \in \mathcal{L}(C, Y)$, we get by (N4.4) and Theorem 7 that

$$|||J + T||| = \sup_{f : \|Uf\|=1} \|U(f + Tf)\| = \sup_{f : \|f\|=1} \|Qf + UTf\| = 1 + |||T|||.$$

■

For the sequel, we need the following

Lemma 9. *Let X, Y be Banach spaces with $X \subset Y$ and $J : X \mapsto Y$ be the natural embedding. Let $V \subset \mathcal{L}(X, Y)$ be a subspace of operators satisfying the Daugavet equation. Then J cannot be represented as a sum of a pointwise unconditionally convergent series of operators from V.*

The proof of this fact is given in [1].

Corollary 10. *Let an unconditional sum Z of Banach spaces X_n, $n = 1, 2, \ldots$ contains C; then there exists an X_{n_0} which contains a copy of C.*

P r o o f. Let $P_n : Z \mapsto X_n$ be the natural projections. Then $Y = \oplus_{n=1}^{\infty} P_n(C)$ is a separable Banach space containing C; denote by $J : C \to Y$ the natural embedding. By Theorem 7, there is an equivalent norm on Y such that $(C, Y) \in DPr$. On the other hand, the series $\sum_{n=1}^{\infty} P_n|_C = J$ is point-wise unconditionally convergent. Hence, by (N4.3) and Lemma 9, there exists a P_{n_0} which is not narrow so that it fixes a copy of C by (N4.2). ■

5. The renorming theorem for $L_1[0, 1]$

Denote by λ the Lebesgue measure on the Borel σ-algebra $\mathcal{B}$ on $[0, 1]$ and by $sign(A)$ the set of functions $f \in L_1$ satisfying

$$|f| = \chi_A, \int_A f d\lambda = 0.$$

Definition 3. *Let Y be a Banach space. We say that an operator $T \in \mathcal{L}(L_1, Y)$ is narrow if, for each $A \in \mathcal{B}$ with $\lambda(A) > 0$ and for each $\varepsilon > 0$ there is an $f \in sign(A)$ such that $\|Tf\| < \varepsilon$.*

We shall use the following simple fact ([3]).

Lemma 11. *Let Y be a Banach space and $T \in \mathcal{L}(L_1, Y)$ be a narrow operator. Then, for each $\varepsilon > 0$, each $A \in \mathcal{B}$, $\lambda(A) > 0$ and $m = 1, 2, \ldots$, there is a partition A', A'' of A with $\lambda(A') = \frac{1}{2^m}\lambda(A)$ such that the function*

$$g = (2^m - 1)\chi_{A'} - \chi_{A''}$$

satisfies $\|Tg\| < \varepsilon$.

The proof consists in m-fold use of Definition 3.

Theorem 12. *$(L_1, m_0(K)) \in DPr$ with respect to narrow operators, where $K = \overline{B}(L_\infty)$.*

P r o o f. Let $T \in \mathcal{L}(L_1, m_0(K))$ be a narrow operator. We fix an $\varepsilon > 0$ and $f \in S(L_1)$ such that $\|Tf\| > \|T\| - \varepsilon$. Without loss of generality, it can be assumed that

$$B = \{s \in K : (Tf)(s) > \|Tf\| - \varepsilon\}$$

is an s.c. set. We consider $f_0 = \sum_{k=1}^n a_k \chi_{A_k} \in S(L_1)$ which approximates f:

$$\|f - f_0\| < \varepsilon. \tag{8}$$

Since T is narrow, using Lemma 11 we get sequences $\{A'_{k,m}\}_{m=1}^\infty$, $\{A''_{k,m}\}_{m=1}^\infty$ such that $A'_{k,m}$, $A''_{k,m}$ form a partition of A_k with $\lambda(A'_{k,m}) = \frac{1}{2^m}\lambda(A_k)$ and we get functions

$$g_{k,m} = (2^m - 1)\chi_{A'_{k,m}} - \chi_{A''_{k,m}}$$

satisfying $\|Tg_{k,m}\| < \varepsilon\lambda(A_k)$, $k = \overline{1, n}$. Put $g_m = \sum_{k=1}^n a_k g_{k,m}$. By straightforward checking, we obtain

$$\|f_0 + g_m\| = 1, \tag{9}$$

$$\|Tg_m\| < \varepsilon. \tag{10}$$

For the s.c. set B, we can find an $s_0 \in K$ and a weak*-neighborhood $U(s_0)$ satisfying the assertion of Lemma 1. Further, note that

$$\lambda(A'_{k,m}) = \frac{1}{2^m}\lambda(A_k) \longrightarrow 0, \text{ as } m \to \infty;$$

therefore the functions

$$s_m(t) = \begin{cases} \operatorname{sign} a_k, & t \in A'_{k,m},\ k = \overline{1, n} \\ s_0(t), & t \notin \cup_{k=1}^n A'_{k,m} \end{cases}$$

weakly$*$ tend to s_0. Hence $s_{m_0} \in U(s_0)$ for some m_0. It can easily be checked that

$$s_{m_0}(f_0 + g_{m_0}) = 1. \tag{11}$$

By the choice of $U(s_0)$, the set

$$A = \{s \in B : |s_{m_0}(f + g_{m_0}) - s(f + g_{m_0})| < \varepsilon,$$
$$\|Q(f + g_{m_0}) + Tf\| \geq |(f + g_{m_0})(s) + (Tf)(s)|\}$$

is non-empty. Fix an $s \in A$; then, by (8) and (11),

$$|1 - s(f + g_{m_0})| < 3\varepsilon.$$

So, by (10),

$$\|Q(f + g_{m_0}) + T(f + g_{m_0})\| \geq |(f + g_{m_0})(s) + (Tf)(s)| - \varepsilon \geq 1 + \|T\| - 6\varepsilon.$$

Finally, by (8) and (9),

$$\|f + g_{m_0}\| \leq \|f - f_0\| + \|f_0 + g_{m_0}\| \leq 1 + \varepsilon,$$

and we obtain

$$(1 + \varepsilon)\|Q + T\| \geq 1 + \|T\| - 6\varepsilon$$

for arbitrary $\varepsilon > 0$. This completes the proof. ■

Theorem 13. *Let Y be a separable Banach space containing L_1 and $J : L_1 \mapsto Y$ be the natural embedding. Then Y can be renormed so that a new norm coincides with the original one on L_1 and $(L_1, Y) \in DPr$ with respect to narrow operators.*

The proof is the same as the argument of Theorem 8.

6. Remarks

- In [4] M. Talagrand proved that the statement of Corollary 10 is not valid for L_1.
- The renorming theorems and their corollaries carry over the case of reach subspaces of C and L_1 (see [1] for definitions). It is not hardly to modify all the proofs.

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Свойство Даугавета для пар банаховых пространств

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Доказывается, что если сепарабельное банахово пространство Y содержит $C[0,1]$, то Y может быть перенормировано так, чтобы пара $(C[0,1], Y)$ обладала свойством Даугавета по отношению к узким операторам. Аналогичний результат доказан для $L_1[0,1]$.

Властивість Даугавета для пар банахових просторів

В.М. Кадець, Р.В. Швидкий

Доводиться, що якщо сепарабельний банахів простір Y містить $C[0,1]$, то Y може бути перенормовано так, щоб пара $(C[0,1], Y)$ мала властивість Даугавета по відношенню до вузьких операторів. Аналогічний результат доведено для $L_1[0,1]$.