Matematicheskaya fizika, analiz, geometriya
2005, v. 12, No. 2, p. 203–217

Global attractor for nonlinear Mindlin-type plates thermoelasticity

T. Fastovska

Department of Mechanics and Mathematics, V.N. Karazin National University
4 Svobody Sq., Kharkov, 61077, Ukraine
E-mail:Tamara.B.Fastovska@univer.kharkov.ua

Received April 19, 2004
Communicated by I.D. Chueshov

A nonlinear initial-boundary value problem with Dirichlet boundary conditions for thermoelastic Mindlin-type plates dynamics equations is considered. It is shown that weak solutions converge asymptotically to a compact global finite-dimensional attractor.

1. Introduction

Let Ω be a bounded domain in $\mathbb{R}^2$. We consider the following thermoelastic problem:

$$\begin{array}{ll} v_{tt} + \beta_0 v_t - \alpha\Delta v - \xi\nabla \mathrm{div} v + \mu_0 v + \gamma\nabla u + \beta\nabla\theta = -\nabla_v\Phi(v_1, v_2), & \\ u_{tt} + \beta_1 u_t - \mu_1\Delta u - \gamma_1 \mathrm{div} v = F(u), & t > 0, x \in \Omega, \\ \theta_t - \eta\Delta\theta + \mathrm{div} v_t = 0, & \\ & \\ v = 0; u = 0; \theta = 0, & t > 0, x \in \partial\Omega, \\ v(\cdot, 0) = v_0; v_t(\cdot, 0) = v_1; u(\cdot, 0) = u_0; & \\ u_t(\cdot, 0) = u_1; \theta(\cdot, 0) = \theta_0, & x \in \Omega, \end{array} \tag{1}$$

where $v(x,t) = (v_1(x,t), v_2(x,t))$ and $u(x,t)$ are respectively the angles of slope of the transverse sections and deflection averaged with respect to the thickness, $\theta(x,t)$ — the variation of the temperature, and $\nabla_v\Phi$ denotes $\begin{pmatrix} \partial_{v_1}\Phi \\ \partial_{v_2}\Phi \end{pmatrix}$.

The parameters α, β_0, β_1, μ_1, ξ, η, μ_0, γ, γ_1 are positive constants.

Such a problem arises from modelling thermoelastic oscillation of plates based on a Mindlin-type assumption on the displacement. Unlike the Kirchhoff's elastic strain-displacement relations this model doesn't neglect the effects of transverse shear forces [1, 2].

Mathematics Subject Classification 2000: 35B41, 35B35.

We consider also the case $\gamma = \gamma_1 = 0$ when the problem is separated into a classical two-dimensional thermoelasticity problem and a wave equation. Thus, our approach also covers the case of 2D nonlinear thermoelasticity.

Linear 2D thermoelasticity with Dirichlet boundary conditions without viscous damping was studied in [3, 4]. It has been proved that in a radially symmetric domain Ω with radially symmetric data the energy decays exponentially.

The paper is organized as follows. In Section 2 we look at the well-posedness in suitable Sobolev spaces. In Section 3 the existence of global compact attractor and it's finite dimensionality will be proved.

2. Well-posedness result

We start by introducing our assumptions and making precise the meanings of a solution for (1). Denote $W(x,t) = (v(x,t), u(x,t))$ and define the space $\mathfrak{W}_T = \{(W,\theta) : (W,\theta) \in L^\infty(0,T;(H_0^1(\Omega))^3) \times (L^\infty(0,T;L^2(\Omega)) \cap L^2(0,T;H_0^1(\Omega))), W_t \in L^\infty(0,T;(L^2(\Omega))^3)\}$.

The norm and the inner product in a space X will be denoted by $\|\cdot\|$ and $(\cdot,\cdot)$ respectively if $X = (L^2(\Omega))^k$, $k = 1,2$ or 3, otherwise by $\|\cdot\|_X$ and $(\cdot,\cdot)_X$.

Define diagonal operators Γ, D, $K : (L^2(\Omega))^3 \to (L^2(\Omega))^3$ by

$$\Gamma = diag\begin{pmatrix}\gamma_1\\ \gamma_1\\ \gamma\end{pmatrix}, \quad D = diag\begin{pmatrix}\beta_0\gamma_1\\ \beta_0\gamma_1\\ \beta_1\gamma\end{pmatrix}, \quad K = diag\begin{pmatrix}\kappa_1\gamma_1\\ \kappa_1\gamma_1\\ \kappa_2\gamma,\end{pmatrix},$$

where parameters κ_1 and κ_2 are positive and will be determined later. The operator $B : D(B) \to (L^2(\Omega))^3$ with the domain $D(B) = H_0^1(\Omega)$ is defined as follows:

$$B\theta = \begin{pmatrix}\beta\gamma_1\partial_1\theta\\ \beta\gamma_1\partial_2\theta\\ 0\end{pmatrix}$$

and the formally adjoint operator $B^* = \begin{pmatrix}-\beta\gamma_1 \mathrm{div}\\ 0\end{pmatrix}$ maps $D(B^*) = (H_0^1(\Omega))^3$ onto $L^2(\Omega)$.

Consider the system (1) under the following assumptions:

(A1) $F \in C^1(\mathbb{R}), \Phi \in C^2(\mathbb{R}^2)$, and there exist $\alpha_i > 0$, $b_i \in \mathbb{R}$, $i = 1,2$, such that

$$\Phi(z_1, z_2) \geq -\alpha_1(z_1^2 + z_2^2) - b_1,$$
$$\mathfrak{F}(z) \leq \alpha_2|z|^2 + b_2,$$

where $\mathfrak{F}(z) = \int\limits_0^z F(\zeta)d\zeta$, $\alpha_1 < \frac{\alpha\lambda_1}{2}$, and $\alpha_2 < \frac{\mu_1\lambda_1}{2}$, ($\lambda_1$ is the smallest eigenvalue of $-\Delta$ with Dirichlet boundary conditions in $(L^2(\Omega))^2$ (or $L^2(\Omega)$)).

(A2) There exist $a_i > 0, i = \overline{1,6}$, and $\epsilon_0 > 0$ such that

$$-a_1\Phi(v) + \nabla_v\Phi(v)\, v \geq a_2|v|^{2+\epsilon_0} - a_3;$$
$$a_4\mathfrak{F}(u) - F(u)\, u \geq a_5|u|^{2+\epsilon_0} - a_6.$$

(A3) There exist $q \geq 0$ and $C > 0$ such that

$$|F'(u)| \leq C(1 + |u|^q);$$
$$|\partial_1^2\Phi(v)| + |\partial_2^2\Phi(v)| + |\partial_1\partial_2\Phi(v)| \leq C(1 + |v|^q).$$

(A4) $(W_0, W_1, \theta_0) \in X_0 = (H_0^1(\Omega))^3 \times (L^2(\Omega))^3 \times L^2(\Omega)$.

Denote $N(W) = (-\gamma_1\nabla_v\Phi(v), \gamma F(u))$. Now we can rewrite the equations (1) in the following way:

$$\Gamma W_{tt} + A_0 W + DW_t + B\theta = N(W),$$
$$\beta\gamma_1\theta_t - \eta\gamma_1\beta\Delta\theta - B^*W_t = 0,$$

where

$$A_0 = \begin{pmatrix} L_1 & -\gamma_1\xi\partial_1\partial_2 & \gamma\gamma_1\partial_1 \\ -\gamma_1\xi\partial_1\partial_2 & L_2 & \gamma\gamma_1\partial_2 \\ -\gamma_1\gamma\partial_1 & -\gamma_1\gamma\partial_2 & L_3 \end{pmatrix},$$

$$L_1 = -\gamma_1\alpha\Delta + \mu_0\gamma_1 - \gamma_1\xi\partial_1^2; L_2 = -\gamma_1\alpha\Delta + \mu_0\gamma_1 - \gamma_1\xi\partial_2^2; L_3 = -\mu_1\gamma\Delta.$$

To prove Lemmas 1 and 3 below we add KW to the both sides of the first equation and rewrite the problem (1) in the following way:

$$\begin{aligned} &\Gamma W_{tt} + AW + DW_t + B\theta = N(W) + KW, && \\ &\beta\gamma_1\theta_t - \eta\gamma_1\beta\Delta\theta - B^*W_t = 0, && t > 0, x \in \Omega, \\ &W = 0; \theta = 0, && t > 0, x \in \partial\Omega, \\ &W(\cdot, 0) = W_0; W_t(\cdot, 0) = W_1; \theta(\cdot, 0) = \theta_0, && x \in \Omega, \end{aligned} \tag{2}$$

where $A = A_0 + K$. Here A is a selfadjoint operator with the domain $D(A) = (H^2(\Omega) \cap H_0^1(\Omega))^3$. A direct calculation gives us that $(AW_1, W_2) = (W_1, AW_2)$ for any $W_1, W_2 \in D(A)$. This implies that operator A is symmetric. It is also obvious that $|(AW_1, W_2)| \leq C\|W_1\|_{(H_0^1)^3}\|W_2\|_{(H_0^1)^3}$. Next we prove the positivity of A for κ_i large enough. For $W = (v_1, v_2, u)$ we have

$$\begin{aligned} (AW, W) &\geq \gamma_1\alpha\|\partial_1 v_1\|^2 + \gamma_1\alpha\|\partial_2 v_1\|^2 + \gamma_1\xi\|\partial_1 v_1 + \partial_2 v_2\|^2 \\ &+\gamma_1\alpha\|\partial_2 v_2\|^2 + \gamma_1\alpha\|\partial_1 v_2\|^2 + \gamma_1\gamma(1-\epsilon)\|\partial_1 u\|^2 - \gamma_1\gamma(\tfrac{1}{\epsilon} - 1)\|v_1\|^2 \\ &+\gamma_1\gamma(1-\epsilon)\|\partial_2 u\|^2 - \gamma_1\gamma(\tfrac{1}{\epsilon} - 1)\|v_2\|^2 + (\mu_1 - \gamma_1)\gamma[\|\partial_1 u\|^2 \\ &+\|\partial_2 u\|^2] + (\mu_0 + \kappa_1 - \gamma)\gamma_1[\|v_1\|^2 + \|v_2\|^2] + \kappa_2\gamma\|u\|^2 \geq \gamma_1\alpha\|\nabla v\|^2 \\ &+(\mu_1 - \epsilon\gamma_1)\gamma\|\nabla u\|^2 + (\mu_0 + \kappa_1 - \tfrac{\gamma}{\epsilon})\gamma_1\|v\|^2 + \kappa_2\gamma\|u\|^2. \end{aligned} \tag{3}$$

Choose $\epsilon < \min\{\frac{\mu_1\lambda_1 - 2\alpha_2}{\gamma_1}, \frac{\mu_1}{\gamma_1}\}$. If κ_1 is large enough for $\mu_0 + \kappa_1 - \frac{\gamma}{\epsilon} \geq 0$ to hold, we have

$$(AW, W) \geq C\|W\|^2_{(H_0^1)^3}.$$

Therefore, by the Friedrichs theorem A is a positive selfadjoint operator and we can define $A^{\frac{1}{2}}$ with the domain $D(A^{\frac{1}{2}}) = (H_0^1(\Omega))^3$.

Further we will consider the problem (1) in the form (2).

Definition 1. *By a weak solution for (2) on* $[0,T]$ *we mean an element* $(W,\theta) \in \mathfrak{W}_T$ *such that* $\theta_t \in L_2(0,T;H^{-1}(\Omega))$, $W(\cdot,0) = W_0 = (v_0,u_0)$, *and the relations*

$$\begin{gathered}
-\int_0^T (\Gamma W_t(t) + DW(t), Y_t(t))dt + \int_0^T (A^{\frac{1}{2}}W(t), A^{\frac{1}{2}}Y(t))dt + \int_0^T (B\theta, Y(t))dt \\
= (\Gamma W_1 + DW_0, Y(0)) + \int_0^T (N(W(t)), Y(t))dt + \int_0^T (KW(t), Y(t))dt; \\
-\beta\gamma_1 \int_0^T (\theta(t), \tau_t(t))dt + \eta\gamma_1\beta \int_0^T (\nabla\theta(t), \nabla\tau(t))dt + \int_0^T (W_t(t), B\theta(t))dt \\
= \beta\gamma_1(\theta_0, \tau(0))
\end{gathered}$$

hold for any $(Y,\tau) \in \mathfrak{W}_T$ *such that* $Y(T) = \tau(T) = 0$, $\tau_t \in L^2(0,T;H^{-1}(\Omega))$. *Here* $W_1 = W_t(\cdot,0) = (v_1,u_1)$.

Now we state the result on existence of solutions.

Theorem 1. *Let* **(A1)**, **(A3)**, **(A4)** *be satisfied. Then on any interval* $[0,T]$ *there exists a unique weak solution* (W,θ) *for (2) such that* $W \in C(0,T;(H_0^1(\Omega))^3)$, $W_t \in C(0,T;(L^2(\Omega))^3)$, $\theta \in C(0,T;L^2(\Omega))$. *The weak solution for (2) satisfies the energy equality*

$$\begin{aligned}
E(t) = E(s) - \int_s^t \|D^{\frac{1}{2}}W_t\|^2 - \beta\gamma_1\eta \int_s^t \|\nabla\theta\|^2 \\
+ \int_s^t (KW, W_t) \quad 0 \leq s \leq t \leq T,
\end{aligned} \tag{4}$$

where

$$\begin{aligned}
E(t) = E(W(t), W_t(t), \theta(t)) = \tfrac{1}{2}\|\Gamma^{\frac{1}{2}}W_t\|^2 + \tfrac{1}{2}\|A^{\frac{1}{2}}W\|^2 \\
+ \tfrac{1}{2}\beta\gamma_1\|\theta\|^2 - \gamma \int_\Omega \mathfrak{F}(u)dx + \gamma_1 \int_\Omega \Phi(v)dx.
\end{aligned} \tag{5}$$

P r o o f. We start with some preliminaries. Consider the auxiliary linear problem:

$$
\begin{aligned}
&\Gamma W_{tt} + AW + DW_t + B\theta = G(x,t), && \\
&\beta\gamma_1\theta_t - \eta\gamma_1\beta\Delta\theta - B^* W_t = 0, && t > 0, x \in \Omega, \\
&W = 0; \theta = 0, && t > 0, x \in \partial\Omega, \\
&W(\cdot,0) = W_0; W_t(\cdot,0) = W_1; \theta(\cdot,0) = \theta_0, && x \in \Omega.
\end{aligned}
\tag{6}
$$

Lemma 1. *Let $G(t) \in L^2(0,T;(L^2(\Omega))^3)$ and* **(A4)** *be satisfied, then for every $\kappa_i \in \mathbb{R}$, $i = 1,2$ the problem (6) has a unique weak solution (W,θ) such that $W \in C(0,T;(H_0^1(\Omega))^3)$, $\theta \in C(0,T;L^2(\Omega))$, $W_t \in C(0,T;(L^2(\Omega))^3)$ and the following energy relation*

$$E_0(t) = E_0(s) - \int_s^t \|D^{\frac{1}{2}} W_t\|^2 - \beta\gamma_1\eta \int_s^t \|\nabla\theta\|^2 + \int_s^t (G(t), W_t) \tag{7}$$

is satisfied, where

$$E_0(t) = \frac{1}{2}\|\Gamma^{\frac{1}{2}} W_t\|^2 + \frac{1}{2}\|A^{\frac{1}{2}} W\|^2 + \frac{1}{2}\beta\gamma_1\|\theta\|^2. \tag{8}$$

(b) Moreover, under the assumption $G(t) \equiv 0$ we have that there exist $\kappa_i^* > 0$ such that for every $\kappa_i \geq \kappa_i^*$, $i = 1,2$, the problem (6) generates a linear dynamical system $(X_0, U(t))$ in the space $X_0 = (H_0^1(\Omega))^3 \times (L^2(\Omega))^3 \times L^2(\Omega)$ with exponentially stable evolution operator $U(t)(W_0,W_1,\theta_0) = (W(t),W_t(t),\theta(t))$, i.e., there exist $C_0, c > 0$ such that

$$\|U(t)(W_0,W_1,\theta_0)\|_{X_0} \leq C_0 e^{-ct}\|(W_0,W_1,\theta_0)\|_{X_0}.$$

P r o o f. In order to prove the first statement of the theorem and existence of the semigroup on the space X_0 one can apply the Faedo–Galerkin method (see, e.g., [5–7]). All calculations are just the same as for nonlinear case given below. We will prove here only the exponential stability of the semigroup. Define

$$H(t) = (\Gamma W_t, W) + \frac{1}{2}\|D^{\frac{1}{2}} W\|^2. \tag{9}$$

It is obvious that

$$
\begin{aligned}
&-\tfrac{\gamma_1}{2\beta_0}\|v_t\|^2 - \tfrac{\gamma}{2\beta_1}\|u_t\|^2 \leq H(t) \\
&\leq \tfrac{\gamma_1}{2\beta_0}\|v_t\|^2 + \tfrac{\gamma}{2\beta_1}\|u_t\|^2 + \gamma_1\beta_0\|v\|^2 + \gamma\beta_1\|u\|^2.
\end{aligned}
\tag{10}
$$

If $G(t) \equiv 0$ it follows from (7) that

$$\tfrac{dE_0}{dt} = -\|D^{\frac{1}{2}} W_t\|^2 - \beta\gamma_1\eta\|\nabla\theta\|^2. \tag{11}$$

From (6) we conclude

$$\tfrac{dH}{dt} = \|\Gamma^{\frac{1}{2}}W_t\|^2 - \|A^{\frac{1}{2}}W\|^2 - (B\theta, W). \tag{12}$$

A suitable Lyapunov function is defined by $V(t) = E_0(t) + \varepsilon H(t)$, where

$$0 < \varepsilon < min\{\beta_0, \beta_1\}. \tag{13}$$

For every $c > 0$ we have from (3), (8), (9), (11), (12)

$$\begin{aligned}\tfrac{dV}{dt} + cV \le -\tfrac{1}{2}(\varepsilon - c)\|A^{\frac{1}{2}}W\|^2 - \gamma_1(\beta_0 - \tfrac{c}{2} - \varepsilon - \tfrac{\varepsilon c}{2})\|v_t\|^2 \\ -\gamma(\beta_1 - \tfrac{c}{2} - \varepsilon - \tfrac{\varepsilon c}{2})\|u_t\|^2 - \beta\gamma_1(\eta - \tfrac{\varepsilon}{2} - \tfrac{c}{2\lambda_1})\|\nabla\theta\|^2 \\ -\tfrac{(\mu_0+\kappa_1-\frac{\gamma}{\epsilon}-\beta_0 c-c-\beta)\varepsilon\gamma_1}{2}\|v\|^2 - \tfrac{(\kappa_2-\beta_1 c-c)\varepsilon\gamma}{2}\|u\|^2.\end{aligned}$$

Choosing ε sufficiently small and as in (13), c such that $\varepsilon > c$ and κ_1, κ_2 large enough, we get

$$\frac{dV}{dt} + cV \le 0.$$

Using (10), we conclude that there exists $C_0 > 0$ such that

$$E_0(t) \le C_0 e^{-ct} E_0(0).$$

This proves the lemma.

Let $\{e_i\}$ be the basis of eigenvectors of $-\Delta$ with the Dirichlet boundary conditions in $L^2(\Omega)$ with the corresponding eigenvalues $\lambda_1 \le \lambda_2 \le ...$ and $\{\tilde{e}_i\}$ be the basis of eigenvectors of $-\Delta$ with the Dirichlet boundary conditions in $(L^2(\Omega))^2$ with the corresponding eigenvalues $\lambda_1 \le \lambda_1 \le \lambda_2 \le \lambda_2 \le \lambda_3 \ldots$. Now we define an approximate solution for the problem (2) corresponding to e_i and $\tilde{e}_i$

$$v_m(t) = \sum_{i=1}^{m} g_i(t)\tilde{e}_i; \quad u_m(t) = \sum_{i=1}^{m} f_i(t)e_i; \quad \theta_m(t) = \sum_{i=1}^{m} \omega_i(t)e_i\,,$$

satisfying the following equations:

$$\begin{aligned}\gamma_1(\ddot{v}_m - \alpha\Delta v_m + \beta_0\dot{v}_m - \xi\nabla\text{div}v_m + (\mu_0 + \kappa_1)v_m + \beta\nabla\theta_m, \tilde{e}_i) \\ = -\gamma_1(\nabla_v\Phi(v_m), \tilde{e}_i) + \kappa_1\gamma_1(v_m, \tilde{e}_i), \\ \gamma(\ddot{u}_m - \mu_1\Delta u_m + \beta_1\dot{u}_m - \gamma_1\text{div}v_m + \kappa_2 u_m, e_i) \\ = \gamma(F(u_m), e_i) + \kappa_2\gamma(u_m, e_i), \\ (\beta\gamma_1\dot{\theta}_m - \eta\gamma_1\beta\Delta\theta_m + \text{div}\dot{v}_m, e_i) = 0\end{aligned} \tag{14}$$

for $i = 1, 2, \ldots, m$ and the initial conditions

$$\begin{aligned}(v_m(0), \tilde{e}_i) = (v_0, \tilde{e}_i); \quad (u_m(0), e_i) = (u_0, e_i); \quad (\theta_m(0), e_i) = (\theta_0, e_i); \\ (\dot{v}_m(0), e_i) = (v_1, \tilde{e}_i); \quad (\dot{u}_m(0), e_i) = (\dot{u}_1, e_i); \ \ i = 1, 2, \ldots, m.\end{aligned} \tag{15}$$

Multiplying the first equation by $\dot{g}_i(t)$, the second by $\dot{f}_i(t)$, the third by $\omega_i(t)$, summing with respect to i, adding the equations, and integrating by parts, we obtain

$$E_m(t) = E_m(0) - \beta\gamma_1\eta \int_0^t \|\nabla\theta_m\|^2 - \int_s^t \|D^{\frac{1}{2}}\dot{W}_m\|^2 + \int_0^t (KW_m, \dot{W}_m), \qquad (16)$$

where $W_m = (v_m, u_m)$ and

$$E_m(t) = \frac{1}{2}\|\Gamma^{\frac{1}{2}}\dot{W}_m\|^2 + \frac{1}{2}\|A^{\frac{1}{2}}W_m\|^2 + \frac{1}{2}\beta\gamma_1\|\theta_m\|^2$$
$$-\gamma \int_\Omega \mathfrak{F}(u_m)dx + \gamma_1 \int_\Omega \Phi(v_m)dx.$$

Observing **(A1)** and **(A4)**, we get from (16) that

$$\|W_m\|_{(H_0^1)^3} + \|\dot{W}_m\|^2 + \|\theta_m\|^2 + \int_0^t \|\nabla\theta_m\|^2 d\tau \le C, \qquad t \in [0, T], \qquad (17)$$

where C depends on T and parameters of the problem. This implies that there exist a subsequence (W_{m_j}, θ_{m_j}) and an element $(W = (v, u), \theta)$ such that

$$\begin{aligned} &W_{m_j} \to W \quad \textit{weak} -^* \ \textit{in } L^\infty(0, T; (H_0^1(\Omega))^3), \\ &\dot{W}_{m_j} \to \dot{W} \quad \textit{weak} -^* \ \textit{in } L^\infty(0, T; (L^2(\Omega))^3), \\ &\theta_{m_j} \to \theta \quad \textit{weak} -^* \ \textit{in } L^\infty(0, T; L^2(\Omega)), \\ &\theta_{m_j} \to \theta \quad \textit{weakly in } L^2(0, T; H_0^1(\Omega)). \end{aligned} \qquad (18)$$

From the known property of weak limits it follows that a weak limit point satisfies the condition

$$\|W\|^2_{(H_0^1)^3} + \|\dot{W}\|^2 + \|\theta\|^2 < C. \qquad (19)$$

Lemma 2. *For any* $w \in L^2(0, T; (L^2(\Omega))^2), y \in L^2(0, T; L^2(\Omega))$

$$\begin{aligned} &(a) \quad \lim_{j\to\infty} \int_0^T (\nabla_v \Phi(v_{m_j}), w)dt = \int_0^T (\nabla_v \Phi(v), w)dt, \\ &(b) \quad \lim_{j\to\infty} \int_0^T (F(u_{m_j}), y)dt = \int_0^T (F(u), y)dt. \end{aligned} \qquad (20)$$

P r o o f. We show first that $\nabla_v\Phi$ is locally Lipschitz from $(H_0^1)^2$ into $(L^2)^2$. We have for $0<\zeta<1$ that

$$\|\nabla_v\Phi(v_{m_j})-\nabla_v\Phi(v)\|^2 \le C\int_\Omega[(1+|v_{m_j}|^q+|v|^q)^2 \\ (v_{1m_j}-v_1)^2+(1+|v_{m_j}|^q+|v|^q)^2(v_{2m_j}-v_2)^2 \\ +(1+|v_{m_j}|^q+|v|^q)^2((v_{1m_j}-v_1)^2+(v_{1m_j}-v_1)^2)]dx.$$

Using the Cauchy–Schwarz inequality and the embedding $H_0^1(\Omega)\subset L^p(\Omega)$ for $1\le p<\infty$, we majorize the last expression by

$$\|\nabla_v\Phi(v_{m_j})-\nabla_v\Phi(v)\|^2 \le \tilde c(1+\|v_{m_j}\|_{(H_0^1)^2}+\|v\|_{(H_0^1)^2})^{2q}\|v_{m_j}-v\|^2_{(L^{\frac{2}{1-\sigma}})^2},$$

where $0<\sigma<1$. Again by the embedding $H_0^1(\Omega)\subset L^p(\Omega)$ for $1\le p<\infty$ and $H^\sigma(\Omega)\subset L^{\frac{2}{1-\sigma}}(\Omega)$ for $0<\sigma<1$, we have

$$\|\nabla_v\Phi(v_{m_j})-\nabla_v\Phi(v)\|^2 \le \tilde c(1+\|v_{m_j}\|_{(H_0^1)^2}+\|v\|_{(H_0^1)^2})^{2q}\|v_{m_j}-v\|^2_{(H_0^1)^2}$$

and

$$\|\nabla_v\Phi(v_{m_j})-\nabla_v\Phi(v)\|^2 \le \tilde c(1+\|v_{m_j}\|_{(H_0^1)^2}+\|v\|_{(H_0^1)^2})^{2q}\|v_{m_j}-v\|^2_{(H^\sigma)^2}. \quad (21)$$

It follows from (17) by the compactness of embedding $(L^2(0,T;H_0^1(\Omega))\cap H^1(0,T;L^2(\Omega)))\subset L^2(0,T;H^\sigma(\Omega))$ [8] that $v_{m_j}\to v$ strongly in $L^2(0,T;H^\sigma(\Omega))$. Thus, from (17), (19) and (21) we have

$$\|\int_0^T(\nabla_v\Phi(v_m),w)d\tau-\int_0^T(\nabla_v\Phi(v),w)d\tau\| \\ \le C\int_0^T((1+\|v_{m_j}\|_{(H_0^1)^2}+\|v\|_{(H_0^1)^2})^q\|v_{m_j}-v\|_{(H^\sigma)^2}\|w\|)d\tau \\ \le\left(C\int_0^T\|v_{m_j}-v\|^2_{(H^\sigma)^2}\right)^{\frac12}\underset{j\to\infty}{\to}0.$$

The case (b) is treated similarly.

From (14), (15), (18), (20) and the structure of equations in (2) it follows that (W,θ) is a weak solution for (2). One can also show (see [8, 7]) that this function satisfies (2) in the sense of distributions and the energy equality (4).

It is obvious that the solution constructed is also the solution for the linearized problem (6) with $G(t)=N(W)+KW$. From Lemma 2 it follows that $G(t)\in L^2(0,T;(L^2(\Omega))^3)$.

Let $(W^1(t),\theta^1(t))$ and $(W^2(t),\theta^2(t))$ be weak solutions for (2) with initial conditions (W_0^1,W_1^1,θ_0^1) and (W_0^2,W_1^2,θ_0^2), respectively. Then the difference of these solutions is the solution for (6) with the initial conditions $(W_0^1 - W_0^2, W_1^1 - W_1^2, \theta_0^1 - \theta_0^2)$ and $G(t) = N(W^1) - N(W^2) + K(W^1 - W^2)$. Therefore, from (8) and Gronwall's lemma it follows that

$$\|(W^1(t) - W^2(t)\|^2_{(H_0^1)^3} + \|\theta^1(t) - \theta^2(t)\|^2 + \|W_t^1(t) - W_t^2(t)\|^2$$
$$\leq C(\|W_0^1 - W_0^2\|^2_{(H_0^1)^3} + \|\theta_0^1 - \theta_0^2\|^2 + \|W_1^1 - W_1^2\|^2),$$

where C depends on T and $R > 0$ such that $E_0(W_0^1, W_1^1, \theta_0^1) + E_0(W_0^2, W_1^2, \theta_0^2) < R$. Therefore, the problem (2) has a unique solution $(W, \theta) = (v, u, \theta)$ such that $W \in C(0,T; (H_0^1(\Omega))^3)$, $W_t \in C(0,T; (L^2(\Omega))^3)$, $\theta \in C(0,T; L^2(\Omega))$.

R e m a r k 1. Theorem 1 establishes existence of a dynamical system (X_0, S_t) with evolution operator defined by $S_t(W_0, W_1, \theta_0) = (W(t), W_t(t), \theta(t))$.

3. Global attractor

In this section we prove our main result on the existence of a compact global attractor of the dynamical system (X_0, S_t) generated by the problem (2). It is known (see, for example, [5, 7]) that to prove the existence of a compact global attractor it is sufficient to show that a dynamical system is dissipative and asymptotically smooth. We recall the corresponding definitions.

Definition 2.

1. *A bounded closed set $\mathfrak{A}$ is said to be a global attractor of dynamical system (X, S_t) if it is strictly invariant and uniformly attracts all trajectories emanating from the bounded sets, i.e., $S_t\mathfrak{A} = \mathfrak{A}$ and for any bounded set $\mathfrak{B} \subset X$*
$$\lim_{t\to\infty} \sup_{y\in\mathfrak{B}} dist(S_t y, \mathfrak{A}) = 0.$$

2. *A closed set $\mathfrak{C}$ is said to be absorbing if for any bounded set $\mathfrak{B}$ there exists $t_0 = t_0(\mathfrak{B})$ such that $S_t(\mathfrak{B}) \subset \mathfrak{C}$ for all $t \geq t_0$. A dynamical system is said to be dissipative if it possesses a bounded absorbing set.*

3. *A dynamical system (X, S_t) is said to be asymptotically smooth if for any bounded set $\mathfrak{B}$ such that $S_t\mathfrak{B} \subset \mathfrak{B}$ for $t > 0$ there exists a compact set $\mathfrak{K}$ in the closure $\bar{\mathfrak{B}}$ of $\mathfrak{B}$ such that*
$$\lim_{t\to\infty} \sup_{y\in\mathfrak{B}} dist(S_t y, \mathfrak{K}) = 0.$$

Our main result is the following assertion.

Theorem 2. *Under assumptions* **(A1)-(A4)** *the problem (2) possesses a finite-dimensional compact global attractor in the space* X_0*. Moreover,*

$$\mathfrak{A} = \mathbb{M}^u(\mathcal{N}), \tag{22}$$

where $\mathcal{N}$ *is the set of stationary points of the system* (X_0, S_t)*, i.e.,* $\mathcal{N} = \{z \in X : S_t z = z \text{ for all } t \geq 0\}$*. The unstable manifold* $\mathbb{M}^u(\mathcal{N})$ *emanating from the set* $\mathcal{N}$ *is the set of all* $y \in X$ *such that there exists a full trajectory* $\{z(t) : t \in \mathbb{R}\}$ *with the properties*

$$z(0) = y, \qquad \lim_{t\to-\infty} dist_X(z(t), \mathcal{N}) = 0.$$

To prove the theorem we need the following lemma.

Lemma 3. *Let in addition to the hypotheses of Theorem 1 Assumption* **(A2)** *holds. Then the dynamical system* (X_0, S_t) *generated by weak solutions for (2) possesses a bounded positive invariant absorbing set* $\mathcal{B}$*.*

P r o o f. As in Lemma 1 we define a function $V(t) = E(t) + \varepsilon H(t)$ where $H(t)$ is defined by (9) and $E(t)$ is given by (5).

$$\frac{dH}{dt} = \|\Gamma^{\frac{1}{2}} W_t\|^2 - \|A^{\frac{1}{2}} W\|^2 - (B\theta, W)$$

$$+(N(W), W) + \|K^{\frac{1}{2}} W\|^2,$$

$$\frac{dE}{dt} = -\|D^{\frac{1}{2}} W_t\|^2 - \beta\gamma_1\eta\|\nabla\theta\|^2 + (KW, W_t).$$

Observing **(A1)** and **(A2)** for κ_1 and κ_2 large enough, we obtain

$$\begin{aligned}
\tfrac{dV}{dt} + cV \leq &-(\varepsilon - \tfrac{c}{2})\|A^{\frac{1}{2}} W\|^2 - \gamma_1(\beta_0 - \tfrac{c}{2} - 2\varepsilon - \tfrac{c\varepsilon}{2})\|v_t\|^2 \\
&-\gamma(\beta_1 - \tfrac{c}{2} - 2\varepsilon - \tfrac{c\varepsilon}{2})\|u_t\|^2 - \beta\gamma_1(\eta - \tfrac{\varepsilon}{2} - \tfrac{c}{2\lambda_1})\|\nabla\theta\|^2 \\
&+(\tfrac{\varepsilon(\beta + c + \beta_0 c)}{2} + \varepsilon\kappa_1 + \tfrac{\kappa_1^2}{\varepsilon} + \varepsilon a_1\alpha_1 - \alpha_1 c)\gamma_1\|v\|^2 + (\tfrac{\varepsilon(c + c\beta_1)}{2} + \varepsilon\kappa_2 \\
&+\tfrac{\kappa_2^2}{\varepsilon} + \varepsilon a_4\alpha_2 - c\alpha_2)\gamma\|u\|^2 - a_2\varepsilon\gamma_1\|v\|^{2+\epsilon_0} - a_5\gamma\varepsilon\|u\|^{2+\epsilon_0} + \gamma_1 a_3\varepsilon|\Omega| \\
&+\gamma a_6\varepsilon|\Omega| + \gamma_1(\varepsilon a_1 - c)|\Omega| b_1 + \gamma(\varepsilon a_4 - c)|\Omega| b_2 .
\end{aligned}$$

Since $\epsilon_0 > 0$ we can choose $\varepsilon > 0$, $c > 0$, and $M > 0$ such that

$$\frac{dV}{dt} + cV \leq M. \tag{23}$$

It follows from (10) that there exists M_1, $M_2 \geq 0$ such that $V(z(t)) \geq M_1\|z(t)\|^2_{X_0} - M_2$. This inequality and (23) prove the lemma (for details we refer to [5, Theorem 1.4.1]).

To show that the dynamical system (X_0, S_t) is asymptotically smooth we need the following result.

Lemma 4. *Let* $z_1(t) = (W^1(t), W_t^1(t), \theta^1(t))$ *and* $z_2(t) = (W^2(t), W_t^2(t), \theta^2(t))$ *be two solutions for the problem (2) lying in the bounded absorbing set* $\mathcal{B}$ *and*

$$\bar{z}(t) = \begin{pmatrix} W^1(t) - W^2(t) \\ W_t^1(t) - W_t^2(t) \\ \theta^1(t) - \theta^2(t) \end{pmatrix} = \begin{pmatrix} \bar{W}(t) \\ \bar{W}_t(t) \\ \bar{\theta}(t) \end{pmatrix}.$$

Then there exist $C_1, C_2, k_1, k_2 > 0$ *such that*

$$E_0(\bar{z}(T)) \le C_1 e^{-k_1 T} E_0(\bar{z}(0)) + C_2 e^{k_2 T} \max_{0 \le t \le T} \|\bar{W}(t)\|^2 \tag{24}$$

for all $T \ge 0$.

P r o o f. We use the representation of weak solutions for nonlinear problem (2):

$$S_t(\bar{z}(0)) = U(t)\bar{z}(0) + \int_0^t U(t-\tau)(0, (-\gamma_1 \nabla_v \Phi(v^1(\tau)) + \gamma_1 \nabla_v \Phi(v^2(\tau)) \\ + \gamma_1 \kappa_1 \bar{v}(\tau), \gamma F(u_1(\tau)) - \gamma F(u_2(\tau)) + \gamma \kappa_2 \bar{u}(\tau)), 0) d\tau,$$

where $U(t)$ is the evolution operator of the problem (6) with $G \equiv 0$ (see Lemma 1). Observing Lemma 1, we have

$$E_0(\bar{z}(T)) \le c_1 \|S_T(\bar{z}(0))\|^2 \le c_2 e^{-2cT} E_0(\bar{z}(0)) \\ + c_3 (\int_0^T e^{-c(T-\tau)} (\|\nabla_v \Phi(v^1(\tau)) - \nabla_v \Phi(v^2(\tau))\| + \|F(u_1(\tau)) \\ - F(u_2(\tau))\| + \|\bar{W}(\tau)\|) d\tau)^2.$$

Using Cauchy–Schwarz inequality, the interpolation inequality $\|\bar{W}(\tau)\|^2_{(H^\sigma)^3} \le \varepsilon \|\bar{W}\|^2_{(H_0^1)^3} + C(\varepsilon)\|\bar{W}\|^2$ (where $0 < \sigma < 1$ and the perameter $\varepsilon > 0$ can be chosen arbitrary small) and the fact that $\int_0^T e^{-2c(T-\tau)} d\tau = \frac{1}{2c}(1 - e^{-2cT}) \le \frac{1}{2c}$, we get the following estimate:

$$E_0(\bar{z}(T)) \le c_2 e^{-2cT} E_0(\bar{z}(0)) + c_4 \int_0^T e^{-2c(T-\tau)} d\tau \int_0^T (\|\bar{W}(\tau)\|^2_{(H^\sigma)^3} \\ + \|\bar{W}(\tau)\|^2) d\tau \le c_2 e^{-2cT} E_0(\bar{z}(0)) + c_5 \varepsilon \int_0^T E_0(\bar{z}(\tau)) d\tau + c_6 \int_0^T \|\bar{W}(\tau)\|^2 d\tau.$$

Gronwall's lemma yields

$$E_0(\bar{z}(T)) \le c_2 e^{-T(2c - c_5 \varepsilon)} E_0(\bar{z}(0)) + c_6 e^{\varepsilon c_5 T} \int_0^T \|\bar{W}(\tau)\|^2 d\tau. \tag{25}$$

Choosing ε small enough, we conclude from (25) the statement of the lemma.

P r o o f o f T h e o r e m 2. To prove the theorem we use the following result from [9]:

Theorem 3 (I. Chueshov–I. Lasiecka). *Let (X, S_t) be a dynamical system on a complete metric space X endowed with a metric d. Assume that for any bounded positively invariant set B in X there exists $T > 0$, a continuous nondecreasing function $q : \mathbb{R}_+ \to \mathbb{R}_+$ and a pseudometric ρ_B^T on $C(0,T;X)$ such that:*

(i) $q(0) = 0$, $q(s) < s, s > 0$;

(ii) the pseudometric ρ_B^T is precompact (with respect to X) in the following sense: any sequence $\{x_n\} \subset B$ has a subsequence $\{x_{n_k}\}$ such that the sequence $\{y_k\} \subset C(0,T;X)$ of elements $y_k(\tau) = S_\tau x_{n_k}$ is Cauchy with respect to ρ_B^T;

(iii) the following inequality holds

$$d(S_T y_1, S_T y_2) \leq q(d(y_1, y_2)) + \rho_B^T(\{S_\tau y_1\}, \{S_\tau y_2\})$$

for every $y_1, y_2 \in B$, where we denote by $S_\tau y_i$ the element in the space $C(0,T;X)$ given by function $y_i(\tau) = S_\tau y_i$. Then (X, S_t) an asymptotically smooth dynamical system.

We can rewrite (24) as follows

$$\|\bar{z}(T)\|_{X_0} \leq Ce^{-k_1 T}\|\bar{z}(0)\|_{X_0} + \rho^T(\{z_1(t)\}, \{z_2(t)\}),$$

where $\rho^T(\{z_1(t)\}, \{z_2(t)\}) = Ce^{k_2 T} \max\limits_{0 \leq t \leq T} \|\bar{W}(t)\|$ is a precompact pseudometric by the compactness of embedding $(L^2(0,T;(H_0^1(\Omega))^3) \cap H^1(0,T;(L^2(\Omega))^3))$ $\subset C(0,T;(L^2(\Omega))^3)$ and $k_i > 0, i = 1,2$. Select T large enough for $Ce^{-k_1 T}$ to be less then 1. Choosing $X = X_0$, $d(y_1, y_2) = \|y_1 - y_2\|_{X_0}$, $q(s) = Ce^{-k_1 T}s$, we get that the dynamical system (X_0, S_T) is asymptotically smooth and with Lemma 3 this gives [5] existence of a compact global attractor $\mathfrak{A}$.

Now prove that the global attractor has a finite fractal dimension. It follows from (24) that

$$\int\limits_T^{2T} E_0(\bar{z}(t)) \leq \frac{C_1}{k_1} e^{-k_1 T} E_0(\bar{z}(0)) + C_3 \max_{[0,T]} \|\bar{W}(t)\|^2, \tag{26}$$

where $C_3 > 0$. Similar to [10] we define a space $X = X_0 \times X_1$ (where $X_0 = (H_0^1(\Omega))^3 \times (L^2(\Omega))^3 \times L^2(\Omega)$ and $X_1 = (L^2(0,T;(H_0^1(\Omega))^3) \cap H^1(0,T;(L^2(\Omega))^3)) \times L^2(0,T;L^2(\Omega)))$ equipped with the norm

$$\|Z\|^2 = \|z(0)\|_{X_0}^2 + 2\int\limits_0^T E_0(z(t))dt,$$

where $Z = (z(0), z(t))$ and $T > 0$ will be determined later. On the space X we define a compact (by the compactness of the embedding $(L^2(0,T;(H_0^1(\Omega))^3) \cap H^1(0,T;(L^2(\Omega)))^3) \subset C(0,T;(L^2(\Omega))^3))$ seminorm

$$n_T(Z) = \max_{0 \le t \le T} \|W(t)\|.$$

Next we consider the set

$$\mathfrak{A}_T = \{Z = (z(0), z(t)), t \in [0,T] : z(0) \in \mathfrak{A}\}.$$

The operator $V_T : \mathfrak{A}_T \to X$ is defined by the formula $V_T : (z(0), z(t)) \to (z(T), z(t+T))$. Now we will verify that V_T is Lipschitz continuous on $\mathfrak{A}_T$. From (8) and Gronwall's lemma we have

$$E_0(z(t)) \le E_0(z(s)) e^{a_R(t-s)}, \qquad 0 \le s \le t, \tag{27}$$

or after setting $t = T + s$ and integrating from T to $2T$, we get

$$\int_T^{2T} E_0(z(s))ds \le e^{a_R T} \int_0^T E_0(z(s))ds. \tag{28}$$

Observing

$$\frac{1}{2}\|Z_1 - Z_2\|_X^2 = E_0(\bar{z}(0)) + \int_0^T E_0(\bar{z}(t))dt,$$
$$\frac{1}{2}\|V_T Z_1 - V_T Z_2\|_X^2 = E_0(\bar{z}(T)) + \int_T^{2T} E_0(\bar{z}(t))dt,$$

we get from (27) and (28) the Lipschitz property for V_T. From (24) and (26) we conclude that

$$\|V_T Z_1 - V_T Z_2\|_X \le \eta_T \|Z_1 - Z_2\|_X + C_4[n_T(Z_1 - Z_2) + n_T(V_T Z_1 - V_T Z_2)]$$

for all $Z_1, Z_2 \in \mathfrak{A}_T$, where $\eta_T = C_5 e^{-k_1 T}$ and $C_4 > 0$, $C_5 > 0$. We can select T large enough and ε small enough to obtain $\eta_T < 1$. Therefore, all conditions of the Theorem 2.3 in [10] are satisfied and, therefore, $dim_{frac}^X \mathfrak{A}_T < \infty$. This implies that $dim_{frac}^{X_0} \mathfrak{A} < \infty$.

To prove (22) it is enough to show that the functional $\Psi(z(t)) = E(z(t)) - \frac{1}{2}\|K^{\frac{1}{2}} W(t)\|^2$ is the strict Lyapunov function for (X_0, S_t), i.e., it is continuous and $t \mapsto \Phi(S_t y)$ is nonincreasing for any $y \in X_0$ and the equality $\Psi(S_{t_0} y) = \Psi(y)$ for some $t_0 > 0$ and $y \in X_0$ implies that $S_t y = y$ for all $t \ge 0$. Indeed, it is obvious that $\Phi(z)$ is continuous on X_0. From (4) it follows that it is nonincreasing. If $\Psi(z(t_0)) = \Psi(z(0))$ then

$$\int_0^{t_0} \|D^{\frac{1}{2}} W_t\|^2 dt + \int_0^{t_0} \|\nabla\theta\|^2 dt = 0.$$

Therefore, $W_t = 0$ and $\nabla\theta = 0$ for $t \in [0, t_0]$. By the Friedrichs inequality $\theta \equiv 0$. Hence, $W(t) = \tilde{W}$, $\theta(t) = 0$ is the stationary solution for (2). Now the existence of a strict Lyapunov function implies (see, e.g., [5, Theorem 1.6.1]) that $\mathfrak{A} = \mathbb{M}^u(\mathcal{N})$ and Theorem 2 is proved.

R e m a r k 2. In the case $\gamma = \gamma_1 = 0$ corresponding to two-dimensional thermoelasticity we denote $W = v$, operators $\Gamma, D, K : (L^2(\Omega))^2 \to (L^2(\Omega))^2$ are as follows

$$\Gamma = I, \quad D = \beta_0 I, \quad K = 0,$$

where I is the identity on $(L^2(\Omega))^2$, $B\theta = \beta\nabla\theta$ and

$$A_0 = A = \begin{pmatrix} -\alpha\Delta + \mu_0 - \xi\partial_1^2 & -\xi\partial_1\partial_2 \\ -\xi\partial_1\partial_2 & -\alpha\Delta + \mu_0 - \xi\partial_2^2 \end{pmatrix}.$$

A_0 is a selfadjoint operator with the domain $D(A_0) = (H^2(\Omega) \cap H_0^1(\Omega))^2$ and the estimate $(A_0 v, v) \geq \alpha\|\nabla v\|^2 + \mu_0\|v\|^2 + \xi\|\operatorname{div} v\|^2$ holds. As in Lemma 1 we define a function $V(t) = E_0(t) + \varepsilon H(t)$, where $H(t) = (v, v_t) + \frac{\beta_0}{2}\|v\|^2$, $E_0 = \frac{1}{2}\|v_t\|^2 + \frac{1}{2}\|A_0^{\frac{1}{2}} v\|^2 + \frac{\beta}{2}\|\theta\|^2$ and $0 < \varepsilon < \beta_0$. For every $c > 0$ we have

$$\frac{dV}{dt} + cV \leq -(\beta_0 - \varepsilon - \tfrac{c}{2} - \tfrac{c\varepsilon}{2})\|v_t\|^2 - \tfrac{1}{2}(\varepsilon - c)\|A_0^{\frac{1}{2}} v\|^2$$
$$-\varepsilon(\tfrac{\mu_0}{2} - \tfrac{c}{2} - \tfrac{c\beta_0}{2} - \tfrac{\sqrt{\varepsilon}\beta}{2})\|v\|^2 - \beta(\eta - \tfrac{\sqrt{\varepsilon}}{2} - \tfrac{c}{2\lambda_1})\|\nabla\theta\|^2 \leq 0,$$

which gives us exponential stabilization of the linear semigroup. Similarly, for nonlinear problem we obtain, that for the function $V(t) = E(t) + \varepsilon H(t)$ (where $E(t) = \frac{1}{2}\|v_t\|^2 + \frac{1}{2}\|A_0^{\frac{1}{2}}\|^2 + \frac{\beta}{2}\|\theta\|^2 + \int_\Omega \Phi(v)dx$) the following estimate holds for some $M > 0$:

$$\frac{dV}{dt} + cV \leq -(\beta_0 - \varepsilon - \tfrac{c}{2} - \tfrac{c\varepsilon}{2})\|v_t\|^2 - (\varepsilon - \tfrac{c}{2})\|A_0^{\frac{1}{2}} v\|^2 + \varepsilon(\tfrac{c}{2} + \tfrac{c\beta_0}{2} + \tfrac{\varepsilon\beta}{2} + \alpha_1 a_1$$
$$-\tfrac{\alpha_1 c}{\varepsilon})\|v\|^2 - \beta(\eta - \tfrac{\varepsilon}{2} - \tfrac{c}{2\lambda_1})\|\nabla\theta\|^2 - \varepsilon a_2\|v\|^{2+\epsilon_0} + (\varepsilon a_3 + (\varepsilon a_1 - c)b_1)|\Omega| \leq M.$$

A suitable Lyapunov function for this system is $\Psi(z(t)) = E(z(t))$.

From it follows that Theorem 2 also takes place in the case of two-dimensional thermoelasticity.

References

[1] *N.F. Morozov*, The selected two-dimensional problems of elasticity. LSU Publ., Leningrad (1978). (Russian)

[2] *P. Schiavone and R.J. Tait*, Thermal effects in Mindlin-type plates. — *Q. Jl. Mech. Appl. Math.* (1993), v. 46, pt. 1, p. 27–39.

[3] *S. Jiang, J.E. Muñoz Rivera, and R. Racke*, Asymptotic stability and global existence in thermoelasticity with symmetry. — *Quart. Appl. Math.* (1998), v. 56, p. 259–275.

[4] *S. Jiang and R. Racke*, Evolution equations in thermoelasticity. π Monographs Surveys Pure Appl. Math., Chapman&Hall/CRC, Boca Raton (2000), v. 112.

[5] *I.D. Chueshov*, Introduction to the theory of infinite-dimensional dissipative systems. Acta, Kharkov (1999) (Russian). (Engl. transl.: Acta, Kharkov (2002)).

[6] *J.L. Lions and E. Magenes*, Nonhomogeneous boundary value problems and applications. Springer–Verlag, Berlin, Heidelberg, New York (1972).

[7] *R. Temam*, Infinite-dimensional dynamical systems in Mechanics and Physics. Springer–Verlag, New York (1988).

[8] *J.L. Lions*, Quelques methodes de resolution des problemes aux limites non lineaires. Dunod, Paris (1969).

[9] *I. Chueshov and I. Lasieska*, Long-time behavior of the second order evolution equations with nonlinear damping. SNS, Pisa (2004).

[10] *I. Chueshov, M. Eller, and I. Lasieska*, Finite dimensionality of the attractor for a semilinear wave equation with nonlinear boundary dissipation. — *Comm. PDE* (2004), v. 29, No. 14, p. 1847–1876.