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On singular limit dynamics for a class of retarded nonlinear partial differential equations

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It is shown that there exist solutions for a class of retarded partial differential equations describing the problem of oscillations of a plate in a quasistatic setting. A long-time behaviour of the solutions is studied. The main result is the existence of a finite-dimensional global attractor for a wide domain of system's parameters. The connection between attractors for dynamical and quasistatic cases is investigated.

1. Introduction

This paper is devoted to a problem of nonlinear oscillations of an elastic plate in a potential supersonic gas flow. This problem can be described by a class of retarded quasilinear partial differential equations (PDE)

$$\mu\ddot{u}+\gamma\dot{u}+\Delta^2 u-f\left(\int\limits_{\Omega}|\nabla u(x,t)|^2 dx\right)\Delta u+\rho\frac{\partial u}{\partial x_1}-q(u_t)=p_0(x), x\in\Omega, t>0 \quad (1)$$

with the boundary conditions

$$u|_{\partial\Omega}=\Delta u|_{\partial\Omega}=0. \quad (2)$$

Here Ω is a bounded domain in $R^2, x=(x_1,x_2), \mu,\gamma,\rho$ are positive parameters of the system, $\dot{u}=\frac{\partial u}{\partial t}$, Δ is the Laplace operator. Assumptions on the scalar function f (s) will be given below, depending on the statement we prove. We rely here on the Berger approach to large deflection [1] (in [1] $f(s)$ is a linear function).

The retarded term has the form

$$q(u_t;x) = \frac{1}{2\pi k}\int\limits_{-\infty}^{x_1} d\xi \int\limits_0^{2\pi} d\theta \left[\left(a_\theta \frac{\partial}{\partial x_1} + b_\theta \frac{\partial}{\partial x_2}\right)^2 u\right]^*$$

$$\times \left(\xi, x_2 - \frac{x_1-\xi}{k}\cos\theta, t - \kappa_\theta(x_1-\xi)\right),$$

$$a_\theta = \frac{\nu\sin\theta - 1}{\nu - \sin\theta}, \quad b_\theta = \frac{k\cdot\cos\theta}{\nu-\sin\theta}\ , \quad \kappa_\theta(\xi) = \frac{\xi}{k^2}(\nu - \sin\theta), \tag{3}$$

where $\Psi^*(x)$ is the extention of $\Psi(x)$ by zero outside of Ω, and the parameter $\nu > 1$ represents the gas velocity, $k = \sqrt{\nu^2-1}$. Formula (3) shows that the value of retarded term at time t uses values of $u(s)$ for $s \in (t - t_*, t)$, where $t_* = l(\nu-1)^{-1}$ is a time retardation, l is the length of Ω along x_1 axis. That is why here and below we use the notation $u_t = u_t(\theta) = u(t+\theta), \theta \in (-t_*, 0)$.

The retarded character of the equation (1) requires initial conditions (cf. [2]) in the form

$$u|_{t=0+} = u_o; \quad \dot{u}|_{t=0+} = u_1; \quad u|_{t\in(-t_*,0)} = \varphi(x,t). \tag{4}$$

The investigation of the considered problem was begun in nonretarded setting ($q(u_t) \equiv 0$). The existence and uniqueness theorem have been obtained in [3], the long-time behaviour for one dimensional case investigated by various authors (see, e.g., [4–6] and the references therein). The analysis of influence of potential supersonic flow carried out in [7, 8] leads to the retarded equation (1). The Cauchy problem for (1) have been investigated in [9, 10], where the existence and properties of solutions for (1)–(4) in different spaces were studied. It was proved [9, 10] that system (1)–(4) has an attractor which is upper semi-continuous with respect to system's parameters.

The present paper is focused on a singular limit ($\mu \to 0$) dynamics for (1). The case when the inertial forces are essentially weaker than the resistance ones is called a quasistatic case ($\mu << \gamma$) and we arrive to the following equation by taking $\mu = 0$ in (1):

$$\gamma\dot{u} + \Delta^2 u - f\left(\int\limits_\Omega |\nabla u(x,t)|^2 dx\right)\Delta u + \rho\frac{\partial u}{\partial x_1} - q(u_t) = p_0(x), x\in\Omega, t>0 \tag{5}$$

with the boundary (2) and initial conditions

$$u|_{t=0+} = u_o; \quad u|_{t\in(-t_*,0)} = \varphi(x,t). \tag{6}$$

In this paper we study the long-time dynamics of the problem (5), (6), (2) and establish connection between attractors for dynamical case (1)–(4) and quasistatic one (5), (6), (2).

The paper organized as follows. In Section 2, we give necessary definitions, prove the existence theorem and construct an evolution operator for the system (5), (6), (2). The main results are contained in Section 3, where we prove the existence of an global attractor of finite fractal dimension for problem (5), (6), (2). We also prove that attractor for problem (1)–(4) goes to attractor for problem (5), (6), (2), when $\mu \to 0$.

We rely on methods used in [2, 8] in questions related with retarded type of the system, general methods from [11, 12, 5, 13] (see also [14] and the survey [15]), and methods used in [11, 10, 16] in questions related with dependence with respect to parameters.

2. The existence of strong solutions

In this section we give the definitions of the function spaces we use, prove the existence and uniqueness theorem and construct an evolution operator for problem (5), (6), (2).

Let $\{e_k\}_{k=1}^{\infty}$ be an orthonormal basis of $L^2(\Omega)$ consisting of the eigenfunctions of the Dirichlet problem for Ω :

$$\Delta e_k + \lambda_k e_k = 0 \ ; \quad e_k(x) = 0 \text{ if } x \in \partial\Omega \ , \quad 0 < \lambda_1 \le \lambda_2 \le \dots .$$

We use the following scale of spaces:

$$\mathcal{F}_s = \{u = \sum_{k=1}^{\infty} u_k e_k : \| u \|_s^2 \equiv \sum_{k=1}^{\infty} \lambda_k^s u_k^2 < \infty\}, \quad s \in \Re. \tag{2.1}$$

We denote by $\| \, . \, \|$ and $(.,.)$ the norm and the inner product in $\mathcal{F}_o = L^2(\Omega)$. The next assertion is of importance to us (see [8, 9]):

Lemma 2.1. *If* $u(t) \in L^2(-t_*, T; \mathcal{F}_{2+2\sigma})$*, then*

$$\| q(u_t) \|_{2\sigma}^2 \le C t_* \int_{t-t_*}^{t} \| u(\tau) \|_{2+2\sigma}^2 \, d\tau, \ 0 \le \sigma < \frac{1}{4}, \tag{2.2}$$

and the map $u \to q(u,t)$ *is linear and continuous from* $L^2(-t_*, T; \mathcal{F}_{2+2\sigma})$ *to* $L^2(0,T; \mathcal{F}_{2\sigma})$.

Definition 2.1. *A strong solution of problem (5),(6),(2) on an interval* $[0,T]$ *is a vector-function* $u(t) \in L^\infty(0,T;\mathcal{F}_1) \cap L^2(-t_*,T;\mathcal{F}_2)$ *with derivative* $\dot u(t) \in L^2(0,T;\mathcal{F}_{-2})$ *if* $u(\theta) = \varphi(\theta)$*, for* $\theta \in (-t_*, 0)$*, and the equation (5) is satisfied almost everywhere in* t *on* $[0,T]$ *as an equality in* $\mathcal{F}_{-2}$.

The main result of this section is the existence and uniqueness

Theorem 2.1. *Let $u_0 \in \mathcal{F}_1$, $\varphi \in L^2(-t_*, 0; \mathcal{F}_2)$, $p_0 \in \mathcal{F}_0$ and f be a local Lipschitz and satisfying the condition*

$$\inf \lim_{s\to\infty} f(s) \geq -C_f, \tag{2.3}$$

with some constant C_f. Then the problem (5), (6), (2) has a strong solution on any interval [0,T]. This solution is unique and satisfies the properties

$$u(t) \in C(0,T;\mathcal{F}_1) \cap L^2(0,T;\mathcal{F}_3).$$

P r o o f. Define an approximate solution of (5), (6), (2) of order m:

$$u_m(x,t) = \sum_{k=1}^{m} g_k(t) e_k,$$

$$(\gamma \dot{u}_m + \Delta^2 u_m - f(\| \nabla u_m \|^2)\Delta u_m + \rho \frac{\partial u_m}{\partial x_1} - q(u_m) - p_o, e_i) = 0,$$

$$(u_m(0+), e_i) = (u_o, e_i), (u_m(\tau), e_i) = (\varphi(\tau), e_i), \quad \tau \in [-t_*, 0]. \tag{2.4}$$

Here $i = 1, \ldots, m$ and $g_k(t) \in C^1(0,T;R) \cap L^2_{loc}(-t_*, T; R)$ such that $\dot{g}_k(t)$ is an absolutely continuous function.

The local existence theorem can be proved if we rewrite the corresponding retarded system for $g_k(t)$ in the integral form and use the idea of the proof of Theorem 2.2.1 from [2].

Multiplying (2.4) by $\lambda_i g_i(t)$ and summing these relations for $i = 1, ..., m$, we obtain

$$\gamma(\dot{u}_m, -\Delta u_m) + (\Delta^2 u_m, -\Delta u_m) + f\left(\| \nabla u_m \|^2\right)(\Delta u_m, \Delta u_m)$$

$$+\rho(\frac{\partial u_m}{\partial x_1}, -\Delta u_m) + (-q(u_m), -\Delta u_m) + (p_0, \Delta u_m) = 0.$$

Hence using the Cauchy–Schwartz inequality, the Lipschitz property of f, Lemma 2.1, (2.3) and denoting by $-\Delta_D$ the Laplace operator with Dirichlet boundary conditions, one obtains

$$\frac{\gamma}{2}\frac{d}{dt} \| \nabla u_m \|^2 + \| (-\Delta_D)^{\frac{3}{2}} u_m \|^2 \leq C_1 + C_2 \| \Delta u_m \|^2$$

$$+C_3 t_* \int_{t-t_*}^{t} \| \Delta u(\tau) \|^2 \, d\tau,$$

where constants C_i do not depend on m. Note that

$$C_2 \parallel \Delta u_m \parallel^2 \leq \frac{1}{2} \parallel (-\Delta_D)^{\frac{3}{2}} u_m \parallel^2 + \frac{C_2^2}{2} \parallel \nabla u_m \parallel^2 .$$

So

$$\gamma \frac{d}{dt} \parallel \nabla u_m \parallel^2 + \parallel (-\Delta_D)^{\frac{3}{2}} u_m \parallel^2 \leq 2C_1 + C_2^2 \parallel \nabla u_m \parallel^2$$

$$+ C t_* \int_{-t_*}^{t} \parallel \Delta u(\tau) \parallel^2 d\tau.$$

Using $\parallel \Delta u_m(t) \parallel^2 \leq \frac{1}{\lambda_1} \parallel (-\Delta_D)^{\frac{3}{2}}(t) u_m \parallel^2$, for $t > 0$, and denoting by

$$w(t) \equiv \parallel \nabla u_m(t) \parallel^2 + \int_0^t \parallel (-\Delta_D)^{\frac{3}{2}} u_m(\tau) \parallel^2 d\tau,$$

we obtain

$$\frac{d}{dt} w(t) \leq C_1 \left(1 + \int_{-t_*}^{0} \parallel \Delta u_m(\tau) \parallel^2 d\tau \right) + C_2 w(t).$$

Multiplying it by $e^{-C_2 t}$ and integrating from 0 to t, we have

$$\parallel \nabla u_m(t) \parallel^2 + \int_0^t \parallel (-\Delta_D)^{\frac{3}{2}} u_m(\tau) \parallel^2 d\tau$$

$$\leq C_1 \left(\parallel \nabla u_m(0) \parallel^2 + \int_{-t_*}^{0} \parallel \Delta u_m(\tau) \parallel^2 d\tau + 1 \right) e^{C_2 T},\ t \in [0, T]. \tag{2.5}$$

The estimate (2.5) with conditions $u_0 \in \mathcal{F}_1,\ \varphi \in L^2(-t_*, 0; \mathcal{F}_2)$ give boundedness of a family $\{u_m(t)\}_{m=1}^{\infty}$ in the space $L^\infty(0, T; \mathcal{F}_1) \cap L^2(0, T; \mathcal{F}_3)$, where [0,T] is an interval of local existence. From this we also obtain the continuation of solutions on any interval ([2, Theorem 2.3.1]). Hence the estimate (2.5) holds for all $T > 0$.

Using (2.4) and (2.5), we get $\int_0^T \parallel \dot{u}_m(\tau) \parallel_{-2}^2 d\tau \leq C_T$. Hence, denoting by

$$Z \equiv L^\infty(0, T; \mathcal{F}_1) \cap L^2(0, T; \mathcal{F}_3) \cap L^2(-t_*, T; \mathcal{F}_2) \times L^2(0, T; \mathcal{F}_{-2}),$$

we have $\{(u_m(t); \dot{u}_m(t))\}_{m=1}^{\infty}$, which is bounded in the space Z. Then there exist a function $(u(t); \dot{u}(t)) \in Z$ and a subsequence $\{u_{m_k}\} \subset \{u_m\}$ such that

$$(u_{m_k}; \dot{u}_{m_k}) \qquad \text{*-weak converges to } (u; \dot{u}) \text{ in the space Z.} \tag{2.6}$$

In order to show that any *-weak limit is a strong solution of problem (5), (6), (2) we use standard argument (see, e.g., [13]). Using convergence (2.6), we pass the limit $m \to \infty$ in all linear nonretarded terms. To pass the limit in the nonlinear and the retarded terms we use the local Lipschitz property of f, (2.5), Lemma 2.1 and strong convergence $u_m \to u$ in space $L^2(0,T;\mathcal{F}_2)$, which follows from (2.6) and the U.A. Doubinskii theorem. To show that a strong solution $u(t)$ belongs to $C(0,T;\mathcal{F}_1)$, we rewrite (5) in the form $\gamma\dot{u} + \Delta^2 u = M(t)$. Since

$$M(t) = f(\| \nabla u(t) \|^2)\Delta u(t) - \rho\frac{\partial u(t)}{\partial x_1} - q(u_t) + p_o \in L^2(0,T;\mathcal{F}_{-1}),$$

using standard methods (see, e.g., [17]), we obtain the mentioned continuity property.

Let us prove the uniqueness of the strong solution. Let u_1 and u_2 be two solutions of problem (5), (6), (2). Then for $w(t) = u_1 - u_2 \in L^2(0,T;\mathcal{F}_2)$ we have

$$\gamma\dot{w} + \Delta^2 w + q(w_t) + \rho\frac{\partial w}{\partial x_1}$$

$$= f(\| \nabla u_1 \|^2)\Delta u_1 - f(\| \nabla u_2 \|^2)\Delta u_2 \in L^2(0,T;\mathcal{F}_{-2}). \tag{2.7}$$

Then we can multiply (2.7) by $w(t)$ in $\mathcal{F}_0$ and integrate from 0 to t:

$$\gamma\int_0^t (\dot{w},w)d\tau + \int_0^t \| \Delta w \|^2 \, d\tau - \int_0^t (q(w_\tau),w)d\tau$$

$$= \int_0^t \{f(\| \nabla u_1 \|^2)(\Delta u_1, w) - f(\| \nabla u_2 \|^2)(\Delta u_2, w)\}d\tau \equiv \int_0^t G(s)ds.$$

From the Lipschitz property of f and (2.5) we get $G(s) \le C_T \| \nabla w(s) \|^2$. Using $\| \nabla w(s) \|^2 \le \epsilon \| \Delta w(s) \|^2 + \frac{1}{4\epsilon} \| w(s) \|^2$ and

$$\int_0^t (q(w_\tau),w)d\tau \le Ct_*\epsilon \int_{-t_*}^t \| \Delta w \|^2 \, d\tau + C_\epsilon \int_0^t \| w \|^2 \, d\tau,$$

we have

$$\| w(t) \|^2 \le \| w(0) \|^2 + C_1 \int_{-t_*}^t \| \Delta w \|^2 \, d\tau + C_2 \int_0^t \| w \|^2 \, d\tau.$$

The Gronwall lemma gives the uniqueness of strong solutions and completes the proof of Theorem 2.1.

Theorem 2.1 enables one to define an evolution operator $S_t = S_t^{\nu,\rho}$ in the space

$$H_1 \equiv \mathcal{F}_1 \times L^2(-t_*, 0; \mathcal{F}_2)$$

with the norm

$$|w|_{H_1}^2 \equiv \| \nabla u \|^2 + \int_{-t_*}^{0} \| \Delta\varphi(s) \|^2 \, ds, \qquad w = (u; \varphi),$$

by the formula

$$S_t(u_0; \varphi(s)) \equiv (u(t); u(t+s)), \quad s \in (-t_*, 0), \quad t \geq 0.$$

Here $u(t)$ is the strong solution of problem (5), (6), (2) with initial conditions $(u_0; \varphi(s)) \in H_1$.

The notation $S_t^{\nu,\rho}$ emphasizes that we have different dynamical systems by choosing different values of the parameters ν and ρ. Note that owing to dependence of the interval of retardation $t_* = \frac{l}{\nu-1}$ on the gas velocity ν, we see changes of the phase space H_1 when the parameter ν varies. Below we will choose the phase space $H_1 = H_1(\nu_0)$ for all cases $\nu \geq \nu_0$. In this case the only restriction to $\tau \in [-\frac{l}{\nu-1}, 0]$ of the initial function $\varphi(\tau)$ is used.

3. Long-time dynamics

In this section we prove the existence of a compact global attractor for problem (5), (6), (2) and show that an attractor for problem (1)–(4) goes to the attractor for problem (5), (6), (2) in the case of singular limit $\mu \to 0$.

Define the domain of variation of parameters

$$\mathcal{D}_{(\nu_0,\rho_0)} \equiv [\nu_0, +\infty] \times [0, \rho_0]. \tag{3.1}$$

Definition 3.1[12]. *A closed bounded in H_1 set $\mathcal{U}$ is said to be a global attractor if it is strictly invariant ($S_t\mathcal{U} = \mathcal{U}$ for any $t \geq 0$) such that $\lim_{t\to\infty} h(S_tB, \mathcal{U}) = 0$ for any bounded set $B \subset H_1$. Here we have set*

$$h(B, A) \equiv \sup\{\mathrm{dist}_{H_1}(y, A) : y \in B\}.$$

The main result of this section is

Theorem 3.1. *Let ρ_0 be any fixed number and function f is a local Lipschitz satisfying the conditions*

$$sf(s) - \alpha \int_0^s f(\tau) d\tau \geq -C_{ff} \text{ for some } \alpha > 0, \ C_{ff} > 0, \tag{3.2}$$

$$\lim_{s\to+\infty} \inf f(s) > \frac{\rho_0^2}{\gamma}. \tag{3.3}$$

Then there exists ν_0 large enough such that in $\mathcal{D}_{(\nu_0,\rho_0)}$ the dynamical system $(S_t^{\nu,\rho}, H_1(\nu_0))$ has a compact global attractor $\mathcal{U}$ of finite fractal dimension. The attractor is a bounded set in the space $\mathcal{F}_{4+\epsilon} \times L^\infty(-t_*, 0; \mathcal{F}_{4+\epsilon})$ for any $\epsilon < \frac{1}{2}$.

P r o o f. The proof follows usual plan used for an abstract dissipative dynamical system [11,5] and relies on four lemmas. Lemmas 3.1, 3.2 show the existence of an attractor, the next two Lemmas 3.3 and 3.4 are used to prove the finite dimensionality of the attractor by help of the Ladyzenskaya theorem [14].

Lemma 3.1. *Let f be as in Theorem 3.1. Then for any $\epsilon_1 < \frac{1}{2}, \epsilon_2 > 0, T > 0$ and bounded in H_1 set B there exists constant $C_{\epsilon_i,T}(B)$ such that for any strong solution begun in B we have*

$$\| u(t) \|_{4+\epsilon_1} \leq C_{\epsilon_i,T}(B), \qquad \text{for } t \in (\epsilon_2, T]. \tag{3.4}$$

P r o o f o f L e m m a 3.1. Rewrite (5) in the form

$$\dot{u}(t) + Au(t) + M(u(t); u_t) = 0. \tag{3.5}$$

Here $A \equiv (-\Delta_D)^2$ and

$$M(u(t); u_t) \equiv -f(\| \nabla u(t) \|^2)\Delta u(t) - \rho \frac{\partial u(t)}{\partial x_1} - q(u_t) + p_o.$$

Write the variation of constants formula for solution of (3.5):

$$u(t) = e^{-tA}u_0 - \int_0^t e^{-(t-\tau)A} M(u(\tau); u_\tau) d\tau.$$

Denote $\Lambda_i = \lambda_i^2$ eigenvalues of the operator A, and using estimate (see, e.g., [12])

$$\| A^\alpha e^{-tA} u \| \leq \left(\frac{\alpha - \beta}{t} + \Lambda_1 \right)^{\alpha-\beta} e^{-t\Lambda_1} \| A^\beta u \|,$$

we have

$$\| A^\alpha u(t) \| \leq \| A^\alpha e^{-tA} u_0 \|$$

$$+ \int_0^t \| A^\alpha e^{-(t-\tau)A} M(u(\tau); u_\tau) \| d\tau \leq \left(\frac{\alpha}{t} + \Lambda_1 \right)^\alpha e^{-\Lambda_1 t} \| u_0 \|$$

$$+\int_0^t \left(\frac{\alpha-\beta}{t-\tau}+\Lambda_1\right)^{\alpha-\beta} e^{-\Lambda_1(t-\tau)} \parallel A^{\beta} M(u(\tau);u_\tau) \parallel d\tau\,.$$

Theorem 2.1 gives $\parallel M(u(\tau);u_\tau) \parallel_{-1} \leq C_T(B)$, for $\tau \in [0,T]$. Then the last integral converges with $\beta=-\frac{1}{4}$ and $\alpha-\beta<1$. Hence for $\alpha<\frac{3}{4}$ we obtain

$$\parallel A^{\alpha} u(t) \parallel = \parallel (\Delta_D^2)^{\alpha} u(t) \parallel = \parallel u(t) \parallel_{4\alpha} \leq C_{\epsilon_i,T},\ \ \alpha<\frac{3}{4}.$$

Now taking initial time moment $t_0>0$ instead of 0, we can repeat the consideration above using obtained smoothness property and complete the proof of Lemma 3.1.

The following lemma gives the property of dissipativeness of dynamical system (S_t,H_1).

Lemma 3.2. *Under conditions on f as in statement of Theorem 3.1 there exist constants ν_0 and $R=R(\rho_0;\nu)$ such that for any bounded in H_1 set B, any $(\nu,\rho)\in\mathcal{D}_{(\nu_0,\rho_0)}$, exists $t_1(B;\nu)$:*

$$\parallel \Delta u(t) \parallel^2 \leq R_2(\rho_0;\nu), \qquad t>t_1(B;\nu),$$

where $u(t)$ is the strong solution of problem (5), (6), (2) with initial conditions $(u(0);u(s))\in B,\ s\in(-t_*,0)$.

P r o o f o f L e m m a 3.2. Multiply (2.4) by $g_i(t)$ and sum for $i=1,...,m$. Then for $u=u_m$ we get

$$\frac{\gamma}{2}\frac{d}{dt} \parallel u \parallel^2 + \parallel \Delta u \parallel^2 + f(\parallel \nabla u \parallel^2) \parallel \nabla u \parallel^2 - (q(u),u) = (p_0,u). \tag{3.6}$$

Multiply (2.4) by $\dot{g}_i$ and sum for $i=1,...,m$.

$$\frac{1}{2}\frac{d}{dt} \parallel \Delta u \parallel^2 + \gamma \parallel \dot{u} \parallel^2 + \frac{1}{2} f(\parallel \nabla u \parallel^2) \frac{d}{dt} \parallel \nabla u \parallel^2$$
$$+\rho(\frac{\partial u}{\partial x_1},\dot{u}) - (q(u),\dot{u}) = (p_0,\dot{u}). \tag{3.7}$$

Note that

$$\frac{1}{2} f(\parallel \nabla u(t) \parallel^2) \frac{d}{dt} \parallel \nabla u(t) \parallel^2 = \frac{1}{4}\frac{d}{dt} F\left(\parallel \nabla u(t) \parallel^2\right),$$

where $F(r)\equiv\int_0^r f(s)ds$. Choose constant C_F such that

$$\Psi(t) \equiv \gamma \parallel u \parallel^2 + \parallel \Delta u \parallel^2 + \frac{1}{2} F\left(\parallel \nabla u \parallel^2\right) + C_F \geq 0.$$

Add (3.6) and (3.7), taking into account the Cauchy-Schwartz inequality,

$$\frac{1}{2}\frac{d}{dt}\Psi(t) + \| \Delta u \|^2 + \frac{\gamma}{4} \| \dot{u} \|^2 \leq -f(\| \nabla u \|^2) \| \nabla u \|^2$$

$$+\epsilon \| u \|^2 + \frac{\rho^2}{\gamma} \| \nabla u \|^2 + \left(\| q(u) \|^2 + \| p_0 \|^2 \right) \left(\frac{1}{2\epsilon} + \frac{1}{\gamma}\right). \tag{3.8}$$

Condition (3.2) gives $F(\| \nabla u \|^2) \leq \frac{C_{ff}}{\alpha} + \frac{1}{\alpha} f(\| \nabla u \|^2) \| \nabla u \|^2$.
Add term $\beta\Psi(t)$ to the both sides of inequality (3.8):

$$\frac{1}{2}\frac{d}{dt}\Psi(t) + \beta\Psi(t) + \frac{\gamma}{4} \| \dot{u} \|^2 + \| \Delta u \|^2$$

$$\leq \| \nabla u \|^2 \left(f(\| \nabla u \|^2) \left[-1 + \frac{\beta}{2\alpha} \right] + \frac{\rho^2}{\gamma} + \frac{\epsilon + \beta\gamma}{\lambda_1} \right)$$

$$+\frac{C_{ff}}{2\alpha} + \left(\| q(u) \|^2 + \| p_0 \|^2 \right) \left(\frac{1}{2\epsilon} + \frac{1}{\gamma}\right).$$

We can see that under condition (3.3) one can choose $\beta > 0$ and ϵ small enough such that the term $\| \nabla u \|^2 \left(f(\| \nabla u \|^2) \left[-1 + \frac{\beta}{2\alpha} \right] + \frac{\rho^2}{\gamma} + \frac{\epsilon+\beta\gamma}{\lambda_1} \right)$ is bounded. Therefore we can find

$$\frac{d}{dt}\Psi(t) + \beta\Psi(t) \leq C_1 + C_2 \| q(u) \|^2 .$$

Using the Gronwall lemma, one can easily check that

$$\Psi(t) \leq C \left(\Psi(t_0) + \int_{t_0 - t_*}^{t_0} \| \Delta u \|^2 \, d\tau \right) \exp\{-(t-t_0)(\beta - C_2 e^{\beta t_*} t_*^2)\} + C.$$

Lemma 3.1 (see (3.4)) gives for any $t_0 \geq t_*$ that

$$\Psi(t_0) + \int_{t_0 - t_*}^{t_0} \| \Delta u \|^2 \, d\tau \leq C_{t_0}(B),$$

for any bounded in H_1 set B. Hence we can choose ν_0 large enough (t_* small enough) such that $\beta - C_2 e^{\beta t_*} t_*^2 \equiv \beta_0 > 0$ and obtain

$$\| \Delta u(t) \|^2 \leq C_{t_0}(B) \exp\{-\beta_0 (t - t_0)\} + R, \quad t \geq t_0 \geq t_*.$$

Lemma 3.2 is proved.

Lemmas 3.1 and 3.2 give that for any ρ_0 there exist $R_s(\rho_0)$ and ν_0 such that

$$||u(t)||^2_{4+\epsilon} \le R_s(\rho_0), \quad \text{for } t \ge t(B;\nu_0), \quad \epsilon < \frac{1}{2}. \tag{3.9}$$

Here $u(t)$ is any strong solution of problem (5), (6), (2) with parameters $(\nu,\rho) \in \mathcal{D}_{(\nu_0,\rho_0)}$ such that $(u(t);u(\tau)) \in B, \tau \in (-t_*,0)$.

Appeal to (5) and use (3.9). It gives

$$||\dot{u}(t)||^2_{\epsilon} \le R'(\rho_0), \quad \text{for } t \ge t(B;\nu_0), \quad \epsilon < \frac{1}{2}.$$

Hence we obtain that for any strong solution begun in bounded in H_1 set B, there exist a compact set K_s and $t(B;\nu_0)$ such that for $t \ge t(B;\nu_0)$

$$(u(t);u(t+s)) \in K_s \subset\subset H_1. \tag{3.10}$$

Here we set

$$K_s \equiv \left((v;\varphi) : ||v||^2_{4+\epsilon} + \text{ess} \sup_{s\in[-t_*,0]} (||\varphi(s)||^2_{4+\epsilon} + ||\dot{\varphi}(s)||^2_{\epsilon}) \le R_s \right), \epsilon < \frac{1}{2}.$$

It means that the dynamical system (S_t, H_1) is dissipative and compact. Hence [11] it has a compact global attractor.

To complete the proof of Theorem 3.1 we have to prove the finiteness of attractor's fractal dimension. We will rely on the Ladyzenskaya theorem [14] and on the approach presented in [9] for dynamical case. We need the following lemmas

Lemma 3.3. *Let u_1 and u_2 are two solutions of problem (5), (6), (2) belonging to the attractor. Then there exist constants C and ξ such that for function $v(t) \equiv u_1(t) - u_2(t)$ we have*

$$||\nabla v(t)||^2 + \int_0^t ||\Delta v(\tau)||^2 d\tau$$

$$\le C \left(||\nabla v(0)||^2 + \int_{-t_*}^0 ||\Delta v(\tau)||^2 d\tau \right) \exp\{2\xi t\}, \quad t > 0. \tag{3.11}$$

The estimate (3.11) gives

Corollary.

$$|S_t w_1 - S_t w_2|_{H_1} \le C e^{\xi t} |w_1 - w_2|_{H_1}, \quad t > 0. \tag{3.12}$$

P r o o f o f L e m m a 3.3. The function $v(t)$ satisfies the equation

$$\gamma\dot{v}+\Delta^2 v - f(||\nabla u_1||^2)\Delta v$$

$$+\left(f(||\nabla u_1||^2)-f(||\nabla u_2||^2)\right)\Delta u_2 - \rho\frac{\partial v}{\partial x_1} - q(v_t)=0.$$

Let us multiply it by $-\Delta v$. Using $||\Delta v_i||^2 \le C$ and Lemma 2.1, we have

$$\frac{\gamma}{2}\frac{d}{dt}||\nabla v||^2 + ||(-\Delta_D)^{\frac{3}{2}}v||^2 \le C^2||\Delta v||^2 + Ct_*\int\limits_{t-t_*}^{t}||\Delta v||^2 d\tau.$$

Going just as in proof of (2.5), we obtain (3.11).

We need several new definitions. Define the space

$$H_{a,\beta}\equiv \mathcal{F}_a\times L^2(-t_*0;\mathcal{F}_{2+2\beta}),\qquad \beta<\frac{1}{4},\qquad 2\beta\le a\le 2+2\beta.$$

Let us define by P_N the orthoprojector in the space $\mathcal{F}_s$ (see (2.1)) onto the subspace spanned by $\{e_1,...,e_N\}$, and we set $Q_N\equiv I-P_N$. Let $\hat{P}_N$ be the orthoprojector in $H_{a,\beta}$ onto the subspace

$$P_N\mathcal{F}_a\times L^2(-t_*0;P_N\mathcal{F}_{2+2\beta}) \text{ and } \hat{Q}_N\equiv I-\hat{P}_N.$$

Consider the trigonometric basis in $L^2(-t_*,0;R):\{\varphi_j(s)\}_{j=0}^{\infty}$:

$$\varphi_o(s)=\sqrt{\frac{1}{t_*}};\varphi_{2k-1}(s)=\sqrt{\frac{2}{t_*}}\sin\frac{2\pi k}{t_*}s,\varphi_{2k}(s)=\sqrt{\frac{2}{t_*}}\cos\frac{2\pi k}{t_*}s,$$

$$k=1,2,\ldots,$$

and the finite dimensional subspace in $H_{a,\beta}$:

$$P_N\mathcal{F}_a\times\ \mathrm{Lin}\{\varphi_j(s)e_k(x):k=1,..,N;j=0,..,M\}.$$

Denote by $\mathcal{P}_{N,M}$ the orthoprojector in $H_{a,\beta}$ on this subspace.

Lemma 3.4. *Let $w_1,w_2\in\mathcal{U}$-attractor of problem (5), (6), (2), then there exist constants $\alpha>0$, $C>0$ and ν large enough such that*

$$|\,(1-\mathcal{P}_{N,M})(S_t w_1 - S_t w_2)\,|_{H_1}$$

$$\le C\left(e^{-\alpha t}+\frac{1}{\lambda_{N+1}^{2\beta}}e^{\xi t}+\frac{\lambda_N^{\frac{5}{2}}}{M+1}e^{\xi t}\right)|\,w_1-w_2\,|_{H_1}\,. \tag{3.13}$$

P r o o f o f L e m m a 3.4. Consider the following linear problem:

$$\gamma\dot{v} + \Delta^2 v - b(t)\Delta v + \rho\frac{\partial v}{\partial x_1} - q(v_t) + \mu v = h(t). \tag{3.14}$$

Note if $u_1(t)$ and $u_2(t)$ are two solutions of (5), (6), (2) then $v(t) = u_1(t) - u_2(t)$ is a solution of (3.14) with $b(t) = f(||\nabla u_1||^2)$ and

$$h(t) = \mu v + \left(f(||\nabla u_1||^2) - f(||\nabla u_2||^2)\right)\Delta u_2. \tag{3.15}$$

Hence we have from Lemma 3.2 that $\sup_{R_+}|b(t)| \le C < \infty$ on the attractor. Let us study the properties of solutions of (3.14) with $h(t) = 0$. Multiply (3.14) by $(-\Delta_D)^a v$:

$$\frac{\gamma}{2}\frac{d}{dt}||(-\Delta_D)^{\frac{a}{2}}v||^2 + ||(-\Delta_D)^{1+\frac{a}{2}}v||^2 + \mu||(-\Delta_D)^{\frac{a}{2}}v||^2$$

$$\le C|((-\Delta_D)^{1+\frac{a}{2}}v, (-\Delta_D)^{\frac{a}{2}}v)| + |((-\Delta_D)^{\beta}q(v), (-\Delta_D)^{a-\beta}v)|.$$

Hence assuming $\beta < \frac{1}{4}, a \le 2 + 2\beta$, using $||(-\Delta_D)^{a-\beta}v||^2 \le C||(-\Delta_D)^{1+\frac{a}{2}}v||^2$ and Lemma 2.1, we have

$$\frac{\gamma}{2}\frac{d}{dt}||(-\Delta_D)^{\frac{a}{2}}v||^2 + (1-2\epsilon)||(-\Delta_D)^{1+\frac{a}{2}}v||^2 + (\mu - \frac{C}{4\epsilon})||(-\Delta_D)^{\frac{a}{2}}v||^2$$

$$\le \frac{Ct_*}{4\epsilon}\int_{t-t_*}^{t}||(-\Delta_D)^{1+\beta}v||^2 d\tau.$$

Take $\alpha \equiv \frac{2}{\gamma}(1-2\epsilon) > 0$ and μ such that $\frac{2}{\gamma}(\mu - \frac{C}{4\epsilon}) \ge \alpha > 0$:

$$\frac{d}{dt}||(-\Delta_D)^{\frac{a}{2}}v||^2 + \alpha||(-\Delta_D)^{1+\frac{a}{2}}v||^2 + \alpha||(-\Delta_D)^{\frac{a}{2}}v||^2$$

$$\le Ct_*\int_{t-t_*}^{t}||(-\Delta_D)^{1+\beta}v||^2 d\tau.$$

Multiply it by $\exp\{\alpha t\}$ and integrate from 0 to t:

$$e^{\alpha t}||(-\Delta_D)^{\frac{a}{2}}v(t)||^2 - ||(-\Delta_D)^{\frac{a}{2}}v(0)||^2 + \alpha\int_0^t e^{\alpha\tau}||(-\Delta_D)^{1+\beta}v||^2 d\tau$$

$$\le Ct_*^2 e^{\alpha t_*}\int_{-t_*}^{t} e^{\alpha\tau}||(-\Delta_D)^{1+\beta}v||^2 d\tau\,.$$

Choose t_* so small $\alpha - Ct_*^2 e^{\alpha t_*} \equiv z > 0$. Hence

$$||(-\Delta_D)^{\frac{a}{2}} v(t)||^2 + z \int_0^t e^{-\alpha(t-\tau)} ||(-\Delta_D)^{1+\beta} v||^2 d\tau$$

$$\leq C \left(||(-\Delta_D)^{\frac{a}{2}} v(0)||^2 + \int_{-t_*}^{0} ||(-\Delta_D)^{1+\beta} v||^2 d\tau \right) \exp\{-\alpha t\} . \tag{3.16}$$

Denote by $U(t,\tau;h,w)$ the evolution family of maps defined as follows:

$$U(t,\tau;h,w) \equiv (u(t); u(t+\theta)) \in H_{a,\beta}, \quad \theta \in (-t_*, 0),$$

where $u(t)$ is the strong solution of problem (3.14) with right-hand side h and initial condition $(u(\tau); u(\tau+\theta)) \equiv w$. If $h \equiv 0$ we set $U(t,\tau;h,w) = U(t,\tau)w$.

The estimate (3.16) gives the existence of constants C and $\alpha > 0$:

$$|U(t,\tau)w|^2_{H_{a,\beta}} \leq Ce^{-\alpha(t-\tau)} |w|^2_{H_{a,\beta}}. \tag{3.17}$$

As in [9], write the variation of constants formula for problem (3.14):

$$U(t,0;h,w) = U(t,0)w + \int_0^t U(t,\tau) X_0 h(\tau) d\tau, \tag{3.18}$$

where

$$[X_0 h(\tau)](\theta) = \begin{cases} h(\tau), & \theta = 0, \\ 0, & \theta \in (-t_*, 0). \end{cases}$$

Apply to the formula (3.18) the projector $\hat{Q}_N$ in $H_1 \equiv H_{1,0}$, using (3.17):

$$|\hat{Q}_N U(t,0;h,w)|_{H_1} \leq Ce^{-\frac{\alpha t}{2}} |w|_{H_1} + \int_0^t |\hat{Q}_N U(t,\tau) X_0 h(\tau)|_{H_1} d\tau.$$

Note that for $\beta < \frac{1}{4}$ and $a = 1 + 2\beta$ we have

$$|\hat{Q}_N x|^2_{H_1} \equiv |\hat{Q}_N x|^2_{H_{1,0}} \leq \frac{1}{\lambda_{N+1}^{2\beta}} |x|^2_{H_{1+2\beta,\beta}}.$$

It gives

$$|\hat{Q}_N U(t,0;h,w)|^2_{H_1} \leq Ce^{-\alpha t} |w|^2_{H_1} + \frac{1}{\lambda_{N+1}^{2\beta}} \int_0^t e^{-\alpha(t-\tau)} ||h(\tau)||^2_{1+2\beta} d\tau. \tag{3.19}$$

Using (3.15), we can show that on the attractor we have $||h(\tau)||^2_{1+2\beta} \leq C||v(t)||^2_2$. From this, Lemma 3.3 and (3.19) we get

$$|\hat{Q}_N U(t,0;h,w)|^2_{H_1} \leq Ce^{-\alpha t}|w|^2_{H_1} + \frac{C}{\lambda^{2\beta}_{N+1}}|w|^2_{H_1} e^{2\xi t}. \tag{3.20}$$

Now consider two solutions of problem (5), (6), (2) belonging to the attractor: $S_t w_l = (u_l(t), u_l(t+\tau))$, $l = 1,2$, and projector $\mathcal{P}_{N,M}$. Evidently,

$$(1 - \mathcal{P}_{N,M})S_t w_l = (Q_N u_l(t); Q_N u_l(t+s)) + \sum_{j=M+1}^{\infty} \int_{-t_*}^{0} \varphi_j(\tau) P_N u_l(t+\tau) d\tau \varphi_j(s)$$

and

$$|(1 - \mathcal{P}_{N,M})(S_t w_1 - S_t w_2)|^2_{H_1} = |\hat{Q}_N (S_t w_1 - S_t w_2)|^2_{H_1}$$

$$+ \sum_{k=1}^{N} \lambda_k^2 \sum_{j=M+1}^{\infty} \{ \int_{-t_*}^{0} \varphi_j(\tau) \, (u_1(t+\tau) - u_2(t+\tau), e_k \,) \, d\tau \}^2. \tag{3.21}$$

Denote the last item $\sum$. As in [9], we have

$$\sum \leq \frac{C_N \lambda_N^5}{(M+1)^2} \int_{-t_*}^{0} \| \, \dot{u}_1(t+\tau) - \dot{u}_2(t+\tau) \, \|^2_{-3} \, d\tau$$

$$\leq \frac{C_N \lambda_N^5}{(M+1)^2} \int_{-t_*}^{0} |S_{t+\tau} w_1 - S_{t+\tau} w_2|^2_{H_1} d\tau.$$

Using Lemma 3.3, we get

$$\sum \leq C \frac{\lambda_N^5}{(M+1)^2} \exp\{2\xi t\} |w_1 - w_2|^2_{H_1}.$$

Using the last estimate and (3.20) and noting that $S_t w_1 - S_t w_2 = U(t,0;$ $h, w_1 - w_2)$, we obtain (3.13). Lemma 3.4 is proved.

Take any $q < 1$. One can easily check that there exist t_0, N_0, M_0 such that

$$|(1 - \mathcal{P}_{N_0,M_0})(S_{t_o} w_1 - S_{t_o} w_2)|_{H_1} \leq q|w_1 - w_2|_{H_1}.$$

Lemma 3.3 implies

$$|\mathcal{P}_{N,M}(S_{t_o} w_1 - S_{t_o} w_2)|_{H_1} \leq l|w_1 - w_2|_{H_1}.$$

The last two inequality allow us apply the Ladyzenskaya theorem on finite dimensionality of invariant sets[14] and complete the proof of Theorem 3.1.

The rest of the paper is devoted to dependence of an attractor with respect to parameters.

It is proved [10] that if the function $f \in C^1(R_+)$, satistying (3.2) and $\lim\limits_{s\to+\infty} \inf f(s) > \frac{\rho_0^2}{\gamma^2}$, then there exist constants ν_0, μ_0 such that for all $(\mu,\nu,\rho) \in (0,\mu_0) \times [\nu_0,+\infty] \times [0,\rho_0]$ the dynamical system $(S_t^{\mu,\nu,\rho}, \mathcal{H}_1(\nu_0))$ governed by solutions of problem (1)–(4) has a finite-dimensional $(\mathcal{H}_1, \mathcal{H}_{1w})$-attractor (for definition see, e.g., [11, 12]). Here

$$\mathcal{H}_1 \equiv \{w = (u_0, u_1, \varphi) \in \mathcal{F}_4 \times \mathcal{F}_2 \times W, \quad u_0 = \varphi(0)\}$$

with

$$W = \Big(v \in L^2(-t_*, 0; \mathcal{F}_4) : \ \dot{v} \in L^2(-t_*, 0; \mathcal{F}_2)\Big).$$

The evolution operator $S_t^{\mu,\nu,\rho}$ is defined as follows:

$$S_t^{\mu,\nu,\rho} w \equiv (u(t); \dot{u}(t); u(t+\tau)) \in \mathcal{H}_1,$$

where $u(t)$ is a strong solution [10] of problem (1)–(4) with initial data $w \in \mathcal{H}_1$.

Using the results obtained above and some estimates from [10], it is possible to prove the following assertion on closeness of attractors for problems (1)–(4) and (5), (6), (2).

Theorem 3.2 *Let $\mathcal{U}_\mu$ be a $(\mathcal{H}_1, \mathcal{H}_{1w})$-attractor for system (1)–(4). Then*

$$\lim_{\mu\to 0} \sup\{dist_{\mathcal{H}}(y, \mathcal{U}_0) : y \in \mathcal{U}_\mu\} = 0, \tag{3.22}$$

where

$$\mathcal{H} \equiv \mathcal{F}_2 \times \mathcal{F}_0 \times L^2(-t_*, 0; \mathcal{F}_2)$$

and

$$\mathcal{U}_0 \equiv ((u_0; u_1; \varphi) : (u_0; \varphi) \in \mathcal{U}, u_1 \equiv -Au_0 - M(u_0; \varphi)).$$

Here $\mathcal{U}$ is an attractor for system (5), (6), (2), and operators A, M defined as in problem (3.5).

P r o o f. In [10] was proved that there exists μ_0 small enough such that for any trajectory of system (1)–(4)

$$S_t^\mu y_0 = (u_\mu(t); \dot{u}_\mu(t); u_\mu(t)),$$

belonging to $(\mathcal{H}_1, \mathcal{H}_{1w})$-attractor $\mathcal{U}_\mu$, and for all $t \in (-\infty, \infty)$ the following estimate holds:

$$\mu \parallel \ddot{u}_\mu(t) \parallel^2 + \parallel \Delta \dot{u}_\mu(t) \parallel^2 + \parallel \Delta^2 u_\mu(t) \parallel^2 \leq R^2_{\mu 0}, \tag{3.23}$$

uniformly in $(0, \mu_0]$. Since $(\mathcal{H}_1, \mathcal{H}_{1w})$-attractor $\mathcal{U}_\mu$ is weakly closed in the space $\mathcal{H}_1$, there exists element $y_\mu = (u_{0\mu}; u_{1\mu}; \varphi_\mu) \in \mathcal{U}_\mu$ such that

$$d(y_\mu) \equiv \mathrm{dist}_{\mathcal{H}}(y_\mu, \mathcal{U}_0) = \sup\{\mathrm{dist}_{\mathcal{H}}(y, \mathcal{U}_0) : y \in \mathcal{U}_\mu\}.$$

Let $y_\mu(t) = (u_\mu(t); \dot{u}_\mu(t); u_\mu(t+\tau)) \in \mathcal{U}_\mu$ be a trajectory of system (1)–(4) such that $y_\mu(0) = y_\mu$. The estimate (3.23) gives existence of a sequence $\{y_{\mu_n}(t)\}$ and an element $y(t) \equiv (u(t); \dot{u}(t); u(t+\tau)) \in L^\infty(-\infty, \infty; \mathcal{H}_1)$ such that for any interval $[a, b]$ sequence $y_{\mu_n}(t)$ converges to $y(t)$ in *-weak topology of the space $L^\infty(a, b; \mathcal{H}_1)$ when $\mu_n \to 0$.

Hence, using the U.A. Doubinskii theorem, one can obtain

$$\lim_{\mu_n \to 0} \max_{t \in [a,b]} \{\parallel u_{\mu_n}(t) - u(t) \parallel_2 + \parallel \dot{u}_{\mu_n}(t) - \dot{u}(t) \parallel\} = 0. \tag{3.24}$$

The estimate (3.23) gives $\mu \parallel \ddot{u}_\mu(t) \parallel \to 0$ when $\mu \to 0$. This fact and strong convergence (3.24) give possibility to pass limit $\mu_n \to 0$ in the equation (1) and obtain that the function $u(t)$ from definition of $y(t)$ is a strong solution of problem (5), (6), (2). This solution is bounded on R and hence belongs to the attractor. Therefore, using standard way (see, e.g., [18, 16]), we obtain (3.22).

R e m a r k. *Theorem 3.2 supplements the assertion proved in [10] and together with Theorem 3.2 [10] garantees upper semicontinuity of attractor* $\mathcal{U}_{\mu,\nu,\rho}$ *in the closed domain of system's parameters*

$$\mathcal{D}^0_{(\mu_0, \nu_0, \rho_0)} \equiv [0, \mu_0] \times [\nu_0, +\infty] \times [0, \rho_0].$$

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О динамике решений при особом переходе в классе нелинейных дифференциальных уравнений в частных производных с запаздыванием

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Показано существование решений для класса дифференциальных уравнений в частных производных с запаздыванием, описывающих задачу колебаний пластины в квазистатической постановке. Изучается асимптотическое поведение этих решений. Основным результатом является существование конечномерного глобального аттрактора для широкой области изменения параметров системы. Изучается связь аттракторов для динамического и квазистатического случаев.

Про динаміку рішень при особливому переході у класі нелінійних диференційних рівнянь у часткових похідних з запізнюванням

Олександр Резуненко

Показано існування рішень для класу диференційних рівнянь у часткових похідних з запізнюванням, що описують задачу коливань пластин у квазістатичному варіанті. Досліджується асимптотична поведінка цих рішень. Головним результатом є існування скінченновимірного глобального атрактора для широкої області змінювання параметрів системи. Досліджується зв'язок між атракторами для динамічного та квазістатичного випадків.