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Dissipative Zakharov system in two-dimensional thin domain

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We study the dissipative case of the Zakharov system with periodic boundary conditions in the two-dimensional thin domain. We prove that this system has a unique strong solution. Our proof is based on the Galerkin method of approximations.

1. Introduction

In this paper we are interested in the long time behavior of solution of dissipative Zakharov system in the two-dimensional thin domain. This system has the form

$$\begin{cases} \frac{1}{\lambda^2} n_{tt} + \alpha n_t + \beta n - \Delta \left(n + |E|^2 \right) = f \\ iE_t + \Delta E - nE + i\gamma E = g \\ n_t(0,x,s) = n_1(x,s),\, n(0,x,s) = n_2(x,s),\, E(0,x,s) = E_0(x,s) \end{cases}, \qquad (1)$$

where $E : \mathbb{R}_x \times \mathbb{R}_s \times \mathbb{R}_t^+ \to \mathbb{C}$ and $n : \mathbb{R}_x \times \mathbb{R}_s \times \mathbb{R}_t^+ \to \mathbb{R}$. The complex function E represents an envelop of the electric field, and n is a fluctuation of the ion density about its equilibrium value. The parameter λ is proportional to the ion acoustic speed. (More precisely see [11]). The damping parameters $\alpha > 0$, $\gamma > 0$, the external forces $f(x)$ and $g(x)$ and parameter $\beta > 0$ are given.

We consider (1) in the rectangle domain

$$\Omega_\varepsilon = \{(x,s) \in [0,1] \times [0,\varepsilon]\},$$

where $\varepsilon > 0$ is a given parameter. Moreover, we assume the periodic boundary conditions for n and E. As in [5] and [1] we change the thin variable $s \to \dfrac{s}{\varepsilon}$.

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Hereafter, our domain Ω_ε will be replaced by the fixed domain $\Omega = [0,1]^2$. Before rewriting system (1) for a new variable we (as [3] and [9]) introduce a new function $m = n_t + \delta n$, where $\delta > 0$ is a fixed parameter which will be chosen later. After this remark we can rewrite our system (1) as

$$\begin{cases} m = n_t + \delta n \\ m_t + (\alpha\lambda^2 - \delta)m - \delta\left(\alpha\lambda^2 - \delta\right)n + \lambda^2 A_\varepsilon\left(n + |E|^2\right) - \beta\lambda^2|E|^2 = \lambda^2 f \\ iE_t - A_\varepsilon E - nE + (\beta + i\gamma)E = g \\ m(0,x,s) = m_0(x,s),\; n(0,x,s) = n_0(x,s),\; E(0,x,s) = E_0(x,s) \end{cases}, \quad (2)$$

where $A_\varepsilon = \beta I - \Delta_\varepsilon = \beta I - \left(\dfrac{\partial^2}{\partial x^2} + \dfrac{1}{\varepsilon^2}\dfrac{\partial^2}{\partial s^2}\right)$.

Let us remark that A_ε with periodic boundary conditions is a selfadjoint positive operator. Therefore we can define a sequence of norms

$$\|u\|_{r,\varepsilon} = \|A_\varepsilon^{r/2}u\|_0,$$

where $\|u\|_0$ is a usual L^2 norm in

$$H^0_{per} = \left\{u \in L^2_{loc}\left(\mathbb{R}^2\right), u(x,s) = u(x+1,s) = u(x,s+1)\right\}.$$

The corresponding inner product in spaces

$$H^r_{per} = \left\{u(x) \in H^r_{loc}\left(\mathbb{R}^2\right), u(x,s) = u(x+1,s) = u(x,s+1)\right\}$$

is defined as

$$(u,v)_{r,\varepsilon} = (A_\varepsilon^{r/2}u, A_\varepsilon^{r/2}v)_0,$$

where $(u,v)_0$ is a standard inner product in H^0_{per} (we also note that $H^r_{per} = \mathbf{D}\left(A_\varepsilon^{r/2}\right)$). As in [9] we also define a product space $\mathcal{E}_r = H^{r-1}_{per} \times H^r_{per} \times \overline{H^{r+1}_{per}}$, where $\overline{H^{r+1}_{per}} = \left\{u(x) + iv(x) | u, v \in H^{r+1}_{per}\right\}$. We note that every element $u(x) \in H^r_{per}$ $\left(\text{or } u(x) \in \overline{H^r_{per}}\right)$ can be represented in the form

$$u(x,s) = \sum_{k,l=-\infty}^{+\infty} u_{k,l} \exp\left\{2\pi i(kx + ls)\right\},$$

where in the real case the Fourier coefficients $u_{k,l}$ possess the property $\overline{u_{k,l}} = u_{-k,-l}$ and

$$\|u\|_r^2 = \sum_{k,l=-\infty}^{+\infty} \lambda_{k,l}^r |u_{k,l}|^2, \text{ where } \lambda_{k,l} = \beta + 4\pi^2k^2 + 4\pi^2\frac{l^2}{\varepsilon^2}.$$

In the one-dimensional case system (2) was studied by several authors. This system with Dirichlet boundary conditions and $\beta = 0$ was considered by I. Flahaut [3]. She proved that problem (2) has a strong solution and generates a dissipative dynamical system wich possesses a weak attractor. This result was improved by O. Goubet and I. Moise [4], who proved that system (2) with Dirichlet boundary conditions possesses a compact global attractor. The case of periodic boundary conditions was considered in [9]. It was proved that in this case system (2) has a unique solution in the energetic space $\mathcal{E}_s$ and generates a dissipative dynamical system. The main result of [9] is the existence of a compact global attractor which belongs to some Gevrey class. In particular, it means that the elements of the attractor are analytic functions of the spatial variable.

In this paper we consider Zakharov system on the two-dimensional thin domain. The main goal of this work is to prove of the existence and uniqueness of a global solution of (2). The proof of this fact is based on the Galerkin method of approximations. The idea of realization of this method is similar to the one in [3] and [9]. But in comparison with the one-dimensional case we have some technical difficulties. The source of this difficulties is the absence of Agmon and Gagliardo–Nirenberg inequalities as in the one-dimensional case. Indeed, in the case of one spatial variable we have $H^{1/4}_{per} \subset L^4$, but in our case we have only $H^{1/2}_{per} \subset L^4$. The same situation one can see with L^∞-norms. These facts imply that we can not use technic of previous works in the two-dimensional (but not thin) rectangle domain.

The key argument of our realization of thin domain technique is the fact that if $\|u\|_0 \le \dfrac{C}{\varepsilon^\nu}$ with some $0 < \nu < 1$, then, taking ε small enough, we can obtain the inequality

$$\|u\|^4_{L^4} \le \kappa \|u\|^2_{1,\varepsilon} + \frac{C}{\varepsilon^{3\nu}}$$

(see Lemma 1 below). There κ is less then some fixed constant, depending on the parameters of the problem.

The main result of the paper is

Theorem 1. *There exist numerical constants κ_j, $j = \overline{1,\,4}$, and there exists ε_0 such that for any $\varepsilon \le \varepsilon_0$ if*

$$\begin{aligned} &\varepsilon^{1/9-0}\|g\|^2_0 \le \gamma\kappa_1; \quad \varepsilon^{1-0}\|g\|^2_{1,\varepsilon} \le \kappa_3; \quad \varepsilon^{3-0}\|g\|^2_{2,\varepsilon} \le \kappa_4; \\ &\varepsilon^{1/3-0}\|f\|^2_{-1,\varepsilon} \le \frac{\alpha\kappa_2}{\lambda^2}; \quad \varepsilon^{1-0}\|f\|^2_0 \le \frac{\alpha\kappa_3}{\lambda^2}; \quad \varepsilon^{3-0}\|f\|^2_{1,\varepsilon} \le \frac{\alpha\kappa_4}{\lambda^2}, \end{aligned} \tag{3}$$

where $\varepsilon^{\nu-0}$ is an any (less then ν) power of ε, then every initial data from

$$\begin{aligned}\mathbb{E}_2 = \{(m,n,E) \in \mathcal{E}_2 : \quad & \varepsilon^{1/3-0}\|m\|^2_{-1,\varepsilon} \le \kappa_2; \qquad \varepsilon^{1-0}\|m\|^2_0 \le \kappa_3;\\ & \varepsilon^{1/3-0}\|n\|^2_0 \le \frac{\kappa_2}{\lambda^2}; \qquad \varepsilon^{1-0}\|n\|^2_{1,\varepsilon} \le \frac{\kappa_3}{\lambda^2};\\ \varepsilon^{1/9-0}\|E\|^2_0 \le \kappa_1; \quad & \varepsilon^{1/3-0}\|E\|^2_{1,\varepsilon} \le \frac{\kappa_2}{2\lambda^2}; \quad \varepsilon^{1-0}\|E\|^2_{2,\varepsilon} \le \frac{\kappa_3}{2\lambda^2};\}\end{aligned} \tag{4}$$

provides a global solution which belongs to $C_b(\mathbb{R}^+, \mathcal{E}_2)$ – the set of the bounded continuous functions.

We note that in [1, Prop. 2.1] there exist conditions of the type (3) and (4). The difficulties of our conditions will be discussed later. The technique of thin domain was applied in [6] for the parabolic equations. The most famous result of this technique is the existence of global solution for Navier–Stokes equations on thin 3D domain (see [8]).

As in corollary of Theorem 1, we prove a global existence of the solution of Zakharov system in the two-dimensional square domain in the case of the small external forces and the initial data.

2. Existence and uniqueness of the solution

2.1. Functional setting

Let us recall without proof two inequalities for our norms:

$$\|u_x\|^2_0 + \frac{1}{\varepsilon^2}\|u_s\|^2_0 \le \|u\|^2_{1,\varepsilon} \tag{5}$$

and

$$\|u_{xx}\|^2_0 + \frac{1}{\varepsilon^2}\|u_{xs}\|^2_0 + \frac{1}{\varepsilon^4}\|u_{ss}\|^2_0 \le \|u\|^2_{2,\varepsilon}, \tag{6}$$

which will be useful for us.

We use the Galerkin method of approximation to prove the existence of the solution of the Zakharov problem. We take

$$\begin{cases} m^N = \sum_{\varepsilon|k|+|l|\le\varepsilon N} m_{N_{k,\,l}}(t)\exp\{2\pi i(kx+ls)\}, & \overline{m_{N_{k,\,l}}} = m_{N_{-k,\,-l}},\\ n^N = \sum_{\varepsilon|k|+|l|\le\varepsilon N} n_{k,\,l}(t)\exp\{2\pi i(kx+ls)\}, & \overline{n_{N_{k,\,l}}} = n_{N_{-k,\,-l}},\\ E^N = \sum_{\varepsilon|k|+|l|\le\varepsilon N} E_{k,\,l}(t)\exp\{2\pi i(kx+ls)\}, & \end{cases} \tag{7}$$

and find (m^N, n^N, E^N) as a solution of the approximate problem:

$$\begin{cases} m^N = n_t^N + \delta n^N \\ m_t^N + (\alpha\lambda^2 - \delta)m^N - \delta\left(\alpha\lambda^2 - \delta\right)n^N + \lambda^2 A_\varepsilon\left(n^N + P_N|E^N|^2\right) \\ \qquad\qquad\qquad\qquad - \beta\lambda^2 P_N|E^N|^2 = \lambda^2 P_N f \quad , \\ iE_t^N - A_\varepsilon E^N - P_N(n^N E^N) + (\beta + i\gamma)E^N = P_N g \\ (m^N, n^N, E^N)(0,x,s) = (P_N m_0,\, P_N n_0,\, P_N E_0)(x,s) \end{cases} \tag{8}$$

where P_N is the orthoprojector, defined by formula

$$P_N u = \sum\nolimits_{\varepsilon|k|+|l|\le\varepsilon N} u_{N_{k,\,l}}(t)\exp\left\{2\pi i(kx + ls)\right\}.$$

We remark that P_N commutes with the operator A^r, so that, in particular means, $P_N A^r E = A^r P_N E$. It is obvious that the system (8) is a system of ordinary differential equations. Therefore, by the standard existence theorem, we obtain that (8) has a unique solution (m^N, n^N, E^N) for $t \in [0, T_N]$. It is easy to see that uniform estimates for the norm of this solution imply that this solution can be extended to $[0, +\infty)$ and $(m^N, n^N, E^N) \in L^\infty(\mathbb{R}^+, \mathcal{E}_2)$. Taking the limit $N \to \infty$, we obtain the existence of the solution (m, n, E) of the problem (2) in $L^\infty(\mathbb{R}^+, \mathcal{E}_2)$.

2.2. Time uniform a priori estimates

First of all we recall the well-known Sobolev inequality for $u \in H^1_{per}(\Omega)$ that will be useful for us:

$$\|u\|^4_{L^4} \le C\|u\|^2_0\left(\|u\|_0 + \|u_x\|_0\right)\left(\|u\|_0 + \|u_s\|_0\right).$$

We will use this inequality as

Lemma 1. *Let $u \in H^1_{per}(\Omega)$ and there exist a constant $\tilde{C}$ such that $\|u(t)\|^2_0 \le \frac{\tilde{C}}{\varepsilon^\nu}$, where $0 < \nu \le 1$. Then for any fixed κ inequality $\varepsilon^{1-\nu}\tilde{C} \le \kappa$ implies*

$$\|u\|^4_{L^4} \le \kappa\|u\|^2_{1,\varepsilon} + \frac{C(\tilde{C}, \kappa)}{\varepsilon^{3\nu}}. \tag{9}$$

R e m a r k. Our technic of the proof of a priori estimates is based on the induction. First of all we prove a priori estimate for $\|E\|_{L^2}$. Taking into account this estimate and Lemma 1, we obtain a new a priori estimate for $\|m\|_{H^{-1}}$, $\|n\|_{L^2}$ and $\|E\|_{H^1}$, and etc. Since we can prove uniqueness of the solution of the problem

(2) only in the space $\mathcal{E}_2$ then we have to use Lemma 1 three times. Let us note that after each using of this Lemma the exponent of $\frac{1}{\varepsilon}$ is multiplied on 3. Since on the last step we need to have the power of $\frac{1}{\varepsilon}$ being less then 1, we conclude that in the first step it is necessary $\varepsilon^{1/9-0}\|E\|_{L^2}^2 \leq C$. The powers of $\frac{1}{\varepsilon}$ in the conditions (3) and (4) in the Theorem 1 was appeared by these arguments.

P r o o f. Indeed, taking into account $\|u(t)\|_0^2 \leq \tilde{C}\varepsilon^{-\nu}$ and (5), we obviously have

$$\|u\|_{L^4}^4 \leq \frac{\tilde{C}}{\varepsilon^\nu}(\frac{\tilde{C}^{1/2}}{\varepsilon^{\nu/2}} + \|u_x\|_0)(\frac{\tilde{C}^{1/2}}{\varepsilon^{\nu/2}} + \|u_s\|_0)$$
$$\leq \frac{\kappa}{2}\|u\|_{1,\varepsilon}^2 + \frac{\kappa}{2}\|u_x\|_0^2 + \frac{\tilde{C}^2}{2\kappa\varepsilon^{2\nu}}\|u_s\|_0^2 + \frac{C(\tilde{C},\kappa)}{\varepsilon^{3\nu}} \leq \kappa\|u\|_{1,\varepsilon}^2 + \frac{C(\tilde{C},\kappa)}{\varepsilon^{3\nu}}.$$

■

Now we can start to prove a priori estimate for the solution of (8). For the sake of simplicity in the next formulas we will drop the subscript N.

Proposition 1. *If* $\varepsilon^{1/9-0}(\|E_0\|_0^2 + \frac{\|g\|_0^2}{\gamma}) \leq C_{0,1}$ *and*

$$\varepsilon^{1/3-0}(\frac{\lambda^2}{\alpha}\|f\|_{-1,\varepsilon}^2 + \|m_0\|_{-1,\varepsilon}^2 + \lambda^2\|n_0\|_0^2 + 2\lambda^2\|E_0\|_{1,\varepsilon}^2) \leq C_{1,1}$$

for the suitable constants $C_{0,1}$ *and* $C_{1,1}$*, then there exist constant* $C_{2,1}$ *such that for the solution of (8) the following estimate holds:*

$$\varepsilon^{1/3-0}\left(\|m(t)\|_{-1,\varepsilon}^2 + \lambda^2\|n(t)\|_0^2 + 2\lambda^2\|E(t)\|_{1,\varepsilon}^2\right) \leq C_{2,1}, \tag{10}$$

where $C_{2,1}$ *does not depend on* ε *and on the number of the Galerkin approximation.*

P r o o f. Let us multiply the third equation of (8) by $2\overline{E}$ and integrate the imaginary part of the resulting equation over Ω. We have

$$\frac{d}{dt}\|E\|_0^2 + 2\gamma\|E\|_0^2 = 2\Im(g,E)_0. \tag{11}$$

Taking into account that

$$2|(g,E)_0| \leq \gamma\|E\|_0^2 + \frac{1}{\gamma}\|g\|_0^2,$$

from the Gronwall lemma we derive that

$$\|E\|_0^2 \le \|E_0\|_0^2\, e^{-\gamma t} + \frac{\|g\|_0^2}{\gamma}(1 - e^{-\gamma t}) \le \|E_0\|_0^2\, e^{-\gamma t} + \frac{\|g\|_0^2}{\gamma}. \tag{12}$$

Now we multiply the third equation of (8) by $-2\overline{E_t} - 2\gamma\overline{E}$ and integrate the real part of the resulting equation over Ω:

$$\begin{gathered}\frac{d}{dt}\left\{\|E\|_{1,\varepsilon}^2 - \beta\|E\|_0^2 + 2\Re(g,E)_0\right\} + 2\gamma\left\{\|E\|_{1,\varepsilon}^2 - \beta\|E\|_0^2 + \Re(g,E)_0\right\} \\ + 2\Re(nE, E_t + \gamma E)_0 = 0.\end{gathered} \tag{13}$$

Taking into account that

$$2\Re(nE, E_t)_0 = \frac{d}{dt}(n, |E|^2)_0 - (m, |E|^2)_0 + \delta(n, |E|^2)_0,$$

we rewrite (13) as

$$\begin{gathered}\frac{d}{dt}\left\{\|E\|_{1,\varepsilon}^2 - \beta\|E\|_0^2 + 2\Re(g,E)_0 + (n,|E|^2)_0\right\} + 2\gamma\left\{\|E\|_{1,\varepsilon}^2 - \beta\|E\|_0^2\right. \\ \left. + \Re(g,E)_0 + (n,|E|^2)_0\right\} - (m,|E|^2)_0 + \delta(n,|E|^2)_0 = 0.\end{gathered} \tag{14}$$

Now we multiply the second equation of (8) by $2A_\varepsilon^{-1}m$ and integrate over Ω.

$$\begin{gathered}\frac{d}{dt}(\|m\|_{-1,\varepsilon}^2 + \lambda^2\|n\|_0^2) + 2(\alpha\lambda^2 - \delta)\|m\|_{-1,\varepsilon}^2 + 2\lambda^2\|n\|_0^2 \\ + 2\delta(\alpha\lambda^2 - \delta)(m,n)_{-1,\varepsilon} + 2\lambda^2(|E|^2, m)_0 - 2\beta\lambda^2(|E|^2, m)_{-1,\varepsilon} = 2\lambda^2(f,m)_{-1,\varepsilon}.\end{gathered} \tag{15}$$

Taking into account that

$$\begin{aligned}2|(f,m)_{-1,\varepsilon}| &\le \frac{\|f\|_{-1,\varepsilon}^2}{\alpha} + \alpha\|m\|_{-1,\varepsilon}^2 \\ |(n,m)_{-1,\varepsilon}| &\le \|m\|_{-1,\varepsilon}\|n\|_{-1,\varepsilon} \le \frac{1}{\sqrt{\beta}}\|m\|_{-1,\varepsilon}\|n\|_0,\end{aligned}$$

it is easy to prove that if $\delta = \min\left\{\frac{\alpha\lambda^2}{5}, \frac{\beta}{2\alpha}\right\}$ then (15) implies that

$$\begin{gathered}\frac{d}{dt}(\|m\|_{-1,\varepsilon}^2 + \lambda^2\|n\|_0^2) + \delta(\|m\|_{-1,\varepsilon}^2 + \lambda^2\|n\|_0^2) + 2\lambda^2(|E|^2, m)_0 \\ \le \frac{\lambda^2}{\alpha}\|f\|_{-1,\varepsilon}^2 + 2\beta\lambda^2(|E|^2, m)_{-1,\varepsilon}.\end{gathered} \tag{16}$$

Let us consider the combination $2\lambda^2(14) + (16)$

$$\begin{gathered}\frac{d}{dt}V_1(t) + \theta V_1(t) + \frac{\delta}{2}(\|m\|_{-1,\varepsilon}^2 + \lambda^2\|n\|_0^2) + 2\gamma\lambda^2\|E\|_{1,\varepsilon}^2 \le \frac{\lambda^2}{\alpha}\|f\|_{-1,\varepsilon}^2 \\ + 2\lambda^2(\delta+\gamma)|(|E|^2, n)_0| + C\left\{|(|E|^2, m)_{-1,\varepsilon}| + |(g,E)_0| + \|E\|_0^2\right\} = R_1(t),\end{gathered} \tag{17}$$

where $\theta = \min\left\{\gamma, \frac{\delta}{2}\right\}$ and

$$V_1(t) = \|m\|^2_{-1,\varepsilon} + \lambda^2\|n\|^2_0 + 2\lambda^2\|E\|^2_{1,\varepsilon} - 2\beta\lambda^2\|E\|^2_0 \\ + 4\lambda^2\Re(g,E)_0 + 2\lambda^2(n,|E|^2)_0. \tag{18}$$

Taking into account that

$$2\lambda^2(\delta+\gamma)|(|E|^2,n)_0| \le \frac{\delta\lambda^2}{2}\|n\|^2_0 + \frac{2\lambda^2(\delta+\gamma)^2}{\delta}\|E\|^4_{L^4}$$

from Lemma 1 (for $\kappa = \frac{\delta\gamma}{2(\delta+\gamma)^2}$), we deduce

$$2\lambda^2(\delta+\gamma)|(|E|^2,n)_0| \le \frac{\delta\lambda^2}{2}\|n\|^2_0 + \gamma\lambda^2\|E\|^2_{1,\varepsilon} + \frac{C}{\varepsilon^{1/3-0}}.$$

Arguing as above, it is easy to obtain

$$R_1(t) \le \frac{\delta}{2}(\|m\|^2_{-1,\varepsilon} + \lambda^2\|n\|^2_0) + 2\gamma\lambda^2\|E\|^2_{1,\varepsilon} + \frac{\lambda^2}{\alpha}\|f\|^2_{-1,\varepsilon} + \frac{C}{\varepsilon^{1/3-0}}. \tag{19}$$

Substituting this estimate for $R_1(t)$ into (17) and using the Gronwall lemma, we obtain

$$V_1(t) \le V_1(0)e^{-\theta t} + \frac{C}{\varepsilon^{1/3-0}}.$$

Similarly to (19) we derive from (18)

$$V_1(t) \ge \frac{\delta}{2}(\|m\|^2_{-1,\varepsilon} + \lambda^2\|n\|^2_0) + \lambda^2\|E\|^2_{1,\varepsilon} - \frac{C}{\varepsilon^{1/3-0}}.$$

The last two formulas imply (10). ■

Proposition 2. *If* $\varepsilon^{1/3-0}C_{2,1} \le C_{1,2}$ *for a suitable constant* $C_{1,2}$ *and*

$$\varepsilon^{1-0}(\frac{\lambda^2}{\alpha}\|f\|^2_0 + \|g\|^2_1 + \|m_0\|^2_0 + \lambda^2\|n_0\|^2_{1,\varepsilon} + 2\lambda^2\|E_0\|^2_{2,\varepsilon}) < C_{2,2}$$

then there exists a constant $C_{2,3}$ *such that for the solution of (8) the following estimate holds:*

$$\varepsilon^{1-0}(\|m(t)\|^2_0 + \lambda^2\|n(t)\|^2_{1,\varepsilon} + 2\lambda^2\|E(t)\|^2_{2,\varepsilon}) \le C_{2,3}, \tag{20}$$

and $C_{2,3}$ *does not depend on* ε *and of the number of the Galerkin approximation.*

P r o o f. Let us multiply the third equation in (8) by $-2\left(A_\varepsilon\overline{E_t}+\gamma A_\varepsilon\overline{E}\right)$, and then integrate the real part of the result over Ω:

$$\frac{d}{dt}\left\{\|E\|^2_{2,\varepsilon}-\beta\|E\|^2_{1,\varepsilon}+2\Re(g,E)_{1,\varepsilon}\right\}+2\gamma\left\{\|E\|^2_{2,\varepsilon}-\beta\|E\|^2_{1,\varepsilon}+\Re(g,E)_{1,\varepsilon}\right\}$$
$$+2\Re(nE,E_t+\gamma E)_{1,\varepsilon}=0. \tag{21}$$

Taking into account that

$$\Re(nE,E_t)_{1,\varepsilon}=\frac{d}{dt}\Re(nE,E)_{1,\varepsilon}-\Re(mE,E)_{1,\varepsilon}+(\delta+\gamma)\Re(nE,E)_{1,\varepsilon}$$
$$+\beta\Im(nE,E)_{1,\varepsilon}-\Im(nP_N(nE),E)_{1,\varepsilon},$$

we rewrite (21) as

$$\frac{d}{dt}\left\{\|E\|^2_{2,\varepsilon}-\beta\|E\|^2_{1,\varepsilon}+2\Re(g,E)_{1,\varepsilon}+2\Re(nE,E)_{1,\varepsilon}\right\}+2\gamma\left\{\|E\|^2_{2,\varepsilon}-\beta\|E\|^2_{1,\varepsilon}\right.$$
$$\left.+\Re(g,E)_{1,\varepsilon}+\Re(nE,E)_{1,\varepsilon}\right\}-2\Re(mE,E)_{1,\varepsilon}-2\Im(P_N(nE),nA_\varepsilon E)_0$$
$$\le C(|(nE,E)_{1,\varepsilon}|+|(g,E)_{1,\varepsilon}|). \tag{22}$$

From the third equation in (8) we can see that

$$nA_\varepsilon E=inE_t-nP_N(nE)+(\beta+i\gamma)nE-nP_Ng.$$

From this equality we obviously have

$$2\Im(P_N(nE),nA_\varepsilon E)_0=-2\Re(P_N(nE),nE_t+n_tE)_0+2\Re(P_N(nE)\bar{E},m)_0$$
$$-2(\gamma+\delta)\Re(P_N(nE),nE)_0-2\Im(P_N(nE),ng)_0=-\frac{d}{dt}\|P_N(nE)\|^2_0$$
$$+2\Re(P_N(nE)\bar{E},m)_0-2(\gamma+\delta)\|P_N(nE)\|^2_0-2\Im(P_N(nE),ng)_0.$$

Substituting this relation into (22), we get

$$\frac{d}{dt}V_{2,1}(t)+2\gamma V_{2,1}(t)-2\Re(mE,E)_{1,\varepsilon}\le R_{2,1}, \tag{23}$$

where

$$V_{2,1}(t)=\|E\|^2_{2,\varepsilon}-\beta\|E\|^2_{1,\varepsilon}+2\Re(g,E)_{1,\varepsilon}+2\Re(nE,E)_{1,\varepsilon}+\|P_N(nE)\|^2_0 \tag{24}$$

and

$$R_{2,1}=|(nE,E)_{1,\varepsilon}|+\|P_N(nE)\|^2_0+\|E\|^2_{1,\varepsilon}+|(P_N(nE)\bar{E},m)_0|$$
$$+|(P_N(nE),ng)_0|+|(g,E)_{1,\varepsilon}|. \tag{25}$$

Now we take the second equation in (8), multiply it by $2m$ and integrate over Ω. As usually we can choose $\delta = \min\left\{\frac{\alpha\lambda^2}{5}, \frac{\beta}{2\alpha}\right\}$, and then we have

$$\begin{aligned} &\frac{d}{dt}\left\{\|m\|_0^2 + \lambda^2\|n\|_{1,\varepsilon}^2\right\} + \delta\left\{\|m\|_0^2 + \lambda^2\|n\|_{1,\varepsilon}^2\right\} + 2\lambda^2\left(m, |E|^2\right)_{1,\varepsilon} \\ &\qquad \le \frac{\lambda^2}{\alpha}\|f\|_0^2 - 2\beta\lambda^2(|E|^2, m)_0. \end{aligned} \tag{26}$$

Taking the combination $(26) + 2\lambda^2(23)$ and choosing $2\theta = \min\{\delta, 2\gamma\}$, we get

$$\begin{aligned} &\frac{d}{dt}\left\{\|m\|_0^2 + \lambda^2\|n\|_{1,\varepsilon}^2 + 2\lambda^2 V_{2,1}(t)\right\} + \theta\left\{\|m\|_0^2 + \lambda^2\|n\|_{1,\varepsilon}^2 + 2\lambda^2 V_{2,1}(t)\right\} \\ &+ \frac{\delta}{2}\|m\|_0^2 + \frac{\delta\lambda^2}{2}\|n\|_{1,\varepsilon}^2 + 2\gamma\lambda^2\|E\|_{2,\varepsilon}^2 \le 2\lambda^2\left(2\Re(mE, E)_{1,\varepsilon} - \left(m, |E|^2\right)_{1,\varepsilon}\right) \\ &\qquad + \frac{\lambda^2}{\alpha}\|f\|_0^2 + 2\beta\lambda^2(|E|^2, m)_0 + C_4 R_{2,1}(t). \end{aligned} \tag{27}$$

Now, using the Cauchy inequality, (5) and (10), we start to estimate the right-hand side of the last inequality as

$$\begin{aligned} &\left|2\Re(mE, E)_{1,\varepsilon} - \left(m, |E|^2\right)_{1,\varepsilon}\right| \le \left|2\Re(mE, E)_{1,\varepsilon} - 2\Re\left(m, \bar{E}A_\varepsilon E\right)_0\right| \\ &+ 2\left|\left(m, |E_x|^2\right)_0\right| + \frac{2}{\varepsilon^2}\left|\left(m, |E_s|^2\right)_0\right| + \beta\left|\left(m, |E|^2\right)_0\right| \\ &\le 2\|m\|_0\left(\|E_x\|_{L^4}^2 + \frac{1}{\varepsilon^2}\|E_s\|_{L^4}^2\right) + \beta\|m\|_0\|E\|_{L^4}^2 \\ &\le \frac{\delta}{6\lambda^2}\|m\|_0^2 + \frac{6\lambda^2}{\delta}\left(\|E_x\|_{L^4}^4 + \frac{1}{\varepsilon^4}\|E_s\|_{L^4}^4\right) + \frac{C}{\varepsilon^{2/3-0}}. \end{aligned}$$

Taking into account Lemma 1 for $u = \frac{1}{\varepsilon}E_s$ and $u = E_x$, we obtain

$$\left|2\Re(mE, E)_{1,\varepsilon} - \left(m, |E|^2\right)_{1,\varepsilon}\right| \le \frac{\delta}{6\lambda^2}\|m\|_0^2 + \frac{\gamma}{3}\|E\|_{2,\varepsilon}^2 + \frac{C}{\varepsilon^{1-0}}.$$

In the same way

$$\begin{aligned} &\left|(|E|^2, m)_0\right| \le \|m\|_0\|E\|_{L^4}^2 \le C\|m\|_0\|E\|_{1,\varepsilon}^2 \le \frac{\delta}{6\beta\lambda^2}\|m\|_0^2 + \frac{C}{\varepsilon^{2/3-0}}; \\ &|(nE, E)_{1,\varepsilon}| \le \|E\|_{2,\varepsilon}\|nE\|_0 \le \|E\|_{2,\varepsilon}\|n\|_{L^4}\|E\|_{L^4} \\ &\le \frac{2\gamma\lambda^2}{3\,C_4}\|E\|_{2,\varepsilon}^2 + \frac{\delta\lambda^2}{8C_4}\|n\|_{1,\varepsilon}^2 + \frac{C}{\varepsilon^{1-0}}; \\ &\|P_N(nE)\|_0^2 \le \|n\|_{L^4}^2\|E\|_{L^4}^2 \le \frac{\delta\lambda^2}{8C_4}\|n\|_{1,\varepsilon}^2 + \frac{C}{\varepsilon^{1-0}}. \end{aligned}$$

And finally

$$|(P_N(nE)\bar{E},m)_0| \le \|m\|_0\|n\|_{L^4}\|E\|_{L^8}^2 \le \frac{\delta}{3\,C_4}\|m\|_0^2 + \frac{\delta\lambda^2}{8C_4}\|n\|_{1,\varepsilon}^2 + \frac{C}{\varepsilon^{1-0}}$$
$$|(P_N(nE),ng)_0| + |(g,E)_{1,\varepsilon}| \le \|g\|_0\|E\|_{2,\varepsilon} + \|g\|_{L^4}\|E\|_{L^4}\|n\|_{L^4}^2$$
$$\le \frac{\delta\lambda^2}{8C_4}\|n\|_{1,\varepsilon}^2 + \frac{2\gamma\lambda^2}{3\,C_4}\|E\|_{2,\varepsilon}^2 + \frac{C}{\varepsilon^{1-0}}.$$

Then we rewrite (27) as

$$\frac{d}{dt}\left\{\|m\|_0^2 + \lambda^2\|n\|_{1,\varepsilon}^2 + 2\lambda^2 V_{2,1}(t)\right\} + \theta\left\{\|m\|_0^2 + \lambda^2\|n\|_{1,\varepsilon}^2 + 2\lambda^2 V_{2,1}(t)\right\}$$
$$+\frac{\delta}{2}\|m\|_0^2 + \frac{\delta\lambda^2}{2}\|n\|_{1,\varepsilon}^2 + 2\gamma\lambda^2\|E\|_{2,\varepsilon}^2$$
$$\le \frac{\delta}{2}\|m\|_0^2 + \frac{\delta\lambda^2}{2}\|n\|_{1,\varepsilon}^2 + 2\gamma\lambda^2\|E\|_{2,\varepsilon}^2 + \frac{C}{\varepsilon^{1-0}}\,.$$

As in Proposition 1 from the Gronwall lemma we get

$$\frac{1}{2}\left(\|m\|_0^2 + \lambda^2\|n\|_{1,\varepsilon}^2 + 2\lambda^2\|E\|_{2,\varepsilon}^2\right) \le \|m\|_0^2 + \lambda^2\|n\|_{1,\varepsilon}^2 + 2\lambda^2 V_{2,1}(t) \le \frac{C_{2,2}}{\varepsilon^{1-0}}\,.$$

■

Proposition 3. *If $\varepsilon^{1-0}C_{2,3} \le C_3$ for a suitable constant C_3 and*

$$\varepsilon^{3-0}\left(\frac{\lambda^2}{\alpha}\|f\|_{1,\varepsilon}^2 + \|g\|_{2,\varepsilon}^2\right) < C$$

then for the solutions of (8) we get

$$\|m(t)\|_{1,\varepsilon}^2 + \lambda^2\|n(t)\|_{2,\varepsilon}^2 + 2\lambda^2\|E(t)\|_{3,\varepsilon}^2 \le \frac{C}{\varepsilon^{3-0}} + C_0e^{-\theta t}, \tag{28}$$

where C_0 depends on initial data and C and C_0 does not depend on ε, and on the number of the Galerkin approximation.

P r o o f. The proof of this proposition is very similar to the proof of the previous ones. But in this proof we can use the fact that in two-dimensional case H^2 is an algebra and $H^{3/2} \subset L^\infty$. In particular,

$$\|u\|_{L^\infty} \le C\|u\|_{1,\varepsilon}^{1/2}\|u\|_{2,\varepsilon}^{1/2}. \tag{29}$$

As in the previous propositions it is easy to obtain that

$$\begin{aligned}&\frac{d}{dt}V_3(t) + \theta V_3(t) + \frac{\delta}{2}\|m\|_{1,\varepsilon}^2 + \frac{\delta\lambda^2}{2}\|n\|_{2,\varepsilon}^2 + 2\gamma\lambda^2\|E\|_{3,\varepsilon}^2\\ &\le 4\lambda^2\Im(nA_\varepsilon E,E)_{2,\varepsilon} + 2\lambda^2\left\{2\Re(mE,E)_{2,\varepsilon} - (|E|^2,m)_{2,\varepsilon}\right\} + C_5R_3(t),\end{aligned} \tag{30}$$

where

$$V_3(t) = \|m\|_{1,\varepsilon}^2 + \lambda^2\|n\|_{2,\varepsilon}^2 + 2\gamma\lambda^2\|E\|_{3,\varepsilon}^2 - 2\beta\lambda^2\|E\|_{2,\varepsilon}^2 + 4\lambda^2\Re(g,E)_{2,\varepsilon} + 4\lambda^2\Re(nE,E)_{2,\varepsilon} \tag{31}$$

and

$$R_3(t) = \frac{\lambda^2\|f\|_{1,\varepsilon}^2}{\alpha} + \|E\|_{2,\varepsilon}^2 + |(|E|^2,m)_{1,\varepsilon}| + |(g,E)_{2,\varepsilon}| + |(nE,E)_{2,\varepsilon}| + |(nP_N(nE),E)_{2,\varepsilon}| + |(ng,E)_{2,\varepsilon}|. \tag{32}$$

Taking into account (5), (6) and (20), we can estimate the terms in the r.h.s of (30) as

$$\Im(nA_\varepsilon E, A_\varepsilon E)_{1,\varepsilon} \le |(n_x A_\varepsilon E, A_\varepsilon E_x)_0| + \frac{1}{\varepsilon^2}|(n_s A_\varepsilon E, A_\varepsilon E_s)_0|$$
$$\le \|E\|_{3,\varepsilon}\|n_x\|_{L^4}\|A_\varepsilon E\|_{L^4} + \frac{1}{\varepsilon}\|E\|_{3,\varepsilon}\|n_s\|_{L^4}\|A_\varepsilon E\|_{L^4}.$$

From Lemma 1 for $u = A_\varepsilon E$, $u = n_x$ and $u = \dfrac{1}{\varepsilon}n_s$ it follows

$$\Im(nA_\varepsilon E, A_\varepsilon E)_{1,\varepsilon} \le \frac{\gamma}{6}\|E\|_{3,\varepsilon}^2 + \frac{\delta}{24}\|n\|_{2,\varepsilon}^2 + \frac{C}{\varepsilon^{3-0}}.$$

For the same reason

$$\left|2\Re(mE,E)_{2,\varepsilon} - (|E|^2,m)_{2,\varepsilon}\right| \le C\|m\|_{L^4}\|E\|_{3,\varepsilon}\|E\|_{2,\varepsilon} + C\|m\|_0\|A_\varepsilon E\|_{L^4}^2$$
$$\le \frac{\delta}{2\,C_5}\|m\|_{1,\varepsilon}^2 + \frac{2\gamma\lambda^2}{3\,C_5}\|E\|_{3,\varepsilon}^2 + \frac{C}{\varepsilon^{3-0}}.$$

We remark that in the above estimate $A_\varepsilon^2|E|^2$ consists from a lot of different terms. But we can divide them into three groups. The first group consists from $\overline{E}A_\varepsilon^2 E + E\,A_\varepsilon^2\overline{E}$. This terms are canceled. The second group consists from the terms which contain the three spatial derivatives on E and one on $\overline{E}$ or vice versa are estimated by $\|m\|_{L^4}\|E\|_{3,\varepsilon}\|E\|_{2,\varepsilon}$. The last group consists from all the rest terms and are estimated by $\|m\|_0\|A_\varepsilon E\|_{L^4}^2$. Then, using Lemma 1 for $u = A_\varepsilon E$, we obtain the above estimate.

Taking into account that $H^1 \subset L^8$, we obtain

$$|(nP_N(nE),E)_{2,\varepsilon}| \le 2\left(\|n_x\|_{L^4} + \tfrac{1}{\varepsilon}\|n_s\|_{L^4}\right)\|n\|_{L^8}\|E\|_{L^8}\|E\|_{3,\varepsilon}$$
$$+ \left(\|E_x\|_{L^4} + \tfrac{1}{\varepsilon}\|E_s\|_{L^4}\right)\|n\|_{L^8}^2\|E\|_{3,\varepsilon} \le \frac{2\gamma\lambda^2}{3\,C_5}\|E\|_{3,\varepsilon}^2 + \frac{\delta\lambda^2}{6\,C_5}\|n\|_{2,\varepsilon}^2 + \frac{C}{\varepsilon^{3-0}}.$$

Using the fact that $H^2(\Omega)$ is an algebra, we can estimate all another terms in the r.h.s of (30) can be estimate as

$$\frac{\lambda^2\|f\|_{1,\varepsilon}^2}{\alpha} + \|E\|_{2,\varepsilon}^2 + |(|E|^2, m)_{1,\varepsilon}| + |(g,E)_{2,\varepsilon}| + |(nE,E)_{2,\varepsilon}| + |(ng,E)_{2,\varepsilon}|$$
$$\leq \frac{\delta\lambda^2}{6\,C_5}\|n\|_{2,\varepsilon}^2 + \frac{C}{\varepsilon^{3-0}}\,,$$

and arguing as in Proposition 2, we get Proposition 3. ■

Taking into account (28), we conclude that the solution of (8) can be extended on the half-axe $[0,+\infty)$. Since there is no difference between limiting transitions (as $N\to\infty$) in our case and the one-dimensional case (see [9]), we omit the proof of this fact. The continuity of the solution of problem (2) (as in previous works (see [3] and [9]) indirectly follows from results in [10] and [7] for Srödinger and wave equation correspondently.

2.3. Uniqueness of the solution

We have proved that problem (2) has a strong solution which belongs to $L^\infty(\mathbb{R}^+,\mathcal{E}_2)$. Let us prove that this solution is unique. Assume the opposite. Let $\left(m^{(1)},n^{(1)},E^{(1)}\right)$ and $\left(m^{(2)},n^{(2)},E^{(2)}\right)$ be two solution of the Zakharov system in $\mathcal{E}_2$. We set $m=m^{(1)}-m^{(2)}$, $n=n^{(1)}-n^{(2)}$ and $E=E^{(1)}-E^{(2)}$. Then (m,n,E) satisfies the following system:

$$\begin{cases} m = n_t + \delta n \\ m_t + \left(\alpha\lambda^2-\delta\right)m - \delta\left(\alpha\lambda^2-\delta\right)n + \lambda^2 A_\varepsilon n = \beta\lambda^2(E^{(1)}\overline{E} + \overline{E^{(2)}}E) \\ -\lambda^2 A_\varepsilon(E^{(1)}\overline{E} + \overline{E^{(2)}}E) \\ iE_t - A_\varepsilon E + (\beta+i\gamma)E = n^{(1)}E + nE^{(2)} \end{cases}. \tag{33}$$

Taking into account that $\left(m^{(k)},n^{(k)},E^{(k)}\right)\in L^\infty\left(\mathbb{R}^+,\mathcal{E}_2\right)$, we note that there exists C_R such that solution $(m(t),n(t),E(t))$ belongs to the ball of radius C_R in $\mathcal{E}_2$, if $\left(m_0^{(k)},n_0^{(k)},E_0^{(k)}\right)$ belong to the ball of radius R in $\mathcal{E}_2$. We note that there exist some constants $C_{1,R}$, $C_{2,R}$ and $C_{3,R}$ such that

$$\begin{aligned} &C_{1,R}\|E\|_{2,\varepsilon} \leq \|E_t\|_0 + \|n\|_0 + \|E\|_0, \\ &\|E_t\|_0 \leq C_{2,R}(\|E\|_0 + \|n\|_0 + \|E\|_{2,\varepsilon}), \\ &C_{3,R}\|E\|_{3,\varepsilon} \leq \|E_t\|_{1,\varepsilon} + \|n\|_{1,\varepsilon} + \|E\|_{1,\varepsilon}. \end{aligned} \tag{34}$$

Let us take the third equation of (33) and differentiate it by t

$$iE_{tt} - A_\varepsilon E_t + (\beta+i\gamma)E_t = n_t^{(1)}E^{(1)} + n^{(1)}E_t^{(1)} - n_t^{(2)}E^{(2)} - n^{(2)}E_t^{(2)}.$$

Then we multiply the imaginary part of this equality on $2A_\varepsilon \bar{E}_t$ and integrate over Ω:

$$\frac{d}{dt}\|E_t\|_{1,\varepsilon}^2 + 2\gamma\|E_t\|_{1,\varepsilon}^2 = 2\Im(n_t E^{(1)} + nE_t^{(1)} + n_t^{(2)} E + n^{(2)} E_t, E_t)_{1,\varepsilon}$$
$$\leq C(\|m\|_{1,\varepsilon}^2 + \|n\|_{2,\varepsilon}^2 + \|E\|_{2,\varepsilon}^2 + \|E_t\|_{1,\varepsilon}^2).$$

Similarly from the third equation of (33) follows that

$$\frac{d}{dt}\|E\|_{2,\varepsilon}^2 + 2\gamma\|E\|_{2,\varepsilon}^2 \leq C(\|n\|_{2,\varepsilon}^2 + \|E\|_{2,\varepsilon}^2).$$

And finally we obtain from the second equation of (33) that

$$\frac{d}{dt}(\|m\|_{1,\varepsilon}^2 + \lambda^2\|n\|_{2,\varepsilon}^2) + \delta(\|m\|_{1,\varepsilon}^2 + \lambda^2\|n\|_{2,\varepsilon}^2) \leq C(\|m\|_{1,\varepsilon}^2 + \|E\|_{3,\varepsilon}^2).$$

Summing the above inequalities and taking into account (34), we get

$$\frac{d}{dt}(\|m\|_{1,\varepsilon}^2 + \|n\|_{2,\varepsilon}^2 + \|E\|_{2,\varepsilon}^2 + \|E_t\|_{1,\varepsilon}^2) \leq C(\|m\|_{1,\varepsilon}^2 + \|n\|_{2,\varepsilon}^2 + \|E\|_{2,\varepsilon}^2 + \|E_t\|_{1,\varepsilon}^2).$$

Therefore

$$(\|m\|_{1,\varepsilon}^2 + \|n\|_{2,\varepsilon}^2 + \|E\|_{2,\varepsilon}^2 + \|E_t\|_{1,\varepsilon}^2)$$
$$\leq e^{Ct}(\|m(0)\|_{1,\varepsilon}^2 + \|n(0)\|_{2,\varepsilon}^2 + \|E(0)\|_{2,\varepsilon}^2 + \|E_t(0)\|_{1,\varepsilon}^2) = 0.$$

This inequality implies the uniqueness of the solution of (2).

Let us prove now Theorem 1.

P r o o f. Let us define $\kappa_1 = \frac{C_{0,1}}{2}$ and $\kappa_2 = \frac{C_{1,1}}{5}$. If we assume that $\varepsilon \leq 1$ from Proposition 1 we infer (10). Then we assume that $\varepsilon \leq \min\left\{1, \frac{C_{1,2}^3}{C_{2,1}^3}\right\}$ and define $\kappa_3 = \frac{C_{2,2}}{5}$. From Proposition 2 we get (20). Let $\varepsilon_0 = \min\left\{1, \frac{C_{1,2}^3}{C_{2,1}^3}, \frac{C_{2,3}}{C_3}\right\}$. Then Proposition 3 implies that

$$\|m\|_{1,\varepsilon}^2 + \lambda^2\|n\|_{2,\varepsilon}^2 + 2\lambda^2\|E\|_{3,\varepsilon}^2 \leq V_3(0)e^{-\theta t} + \frac{C}{\varepsilon^3}.$$

Taking into account this estimate, we conclude the proof of Theorem 1. ■

R e m a r k. We note that conditions (3) and (4) can be rewrite for the initial domain $\Omega_\varepsilon = [0,1] \times [0,\varepsilon]$ in following way:

$$\|m_0\|_{L^2(\Omega_\varepsilon)}^2 \leq \kappa_3; \quad \|n_0\|_{L^2(\Omega_\varepsilon)}^2 \leq \frac{\kappa_2}{\lambda^2}\varepsilon^{2/3+0}; \quad \|E_0\|_{L^2(\Omega_\varepsilon)}^2 \leq \kappa_1\varepsilon^{8/9+0};$$

$$\|\partial_x n_0\|^2_{L^2(\Omega_\varepsilon)} + \|\partial_s n_0\|^2_{L^2(\Omega_\varepsilon)} \le \frac{\kappa_3}{\lambda^2}; \quad \|\partial_x E_0\|^2_{L^2(\Omega_\varepsilon)} + \|\partial_s E_0\|^2_{L^2(\Omega_\varepsilon)} \le \frac{\kappa_2}{2\lambda^2}\varepsilon^{2/3+0};$$

$$\|\partial_{xx} E_0\|^2_{L^2(\Omega_\varepsilon)} + \|\partial_{xs} E_0\|^2_{L^2(\Omega_\varepsilon)} + \|\partial_{ss} E_0\|^2_{L^2(\Omega_\varepsilon)} \le \frac{\kappa_3}{2\lambda^2};$$

and

$$\|f\|^2_{L^2(\Omega_\varepsilon)} \le \frac{\alpha\kappa_3}{\lambda^2}; \qquad \|g\|^2_{L^2(\Omega_\varepsilon)} \le \kappa_1 \varepsilon^{8/9+0};$$

$$\|\partial_x f\|^2_{L^2(\Omega_\varepsilon)} + \|\partial_s f\|^2_{L^2(\Omega_\varepsilon)} \le \frac{\alpha\kappa_4}{\lambda^2\varepsilon^{2+0}}; \quad \|\partial_x g\|^2_{L^2(\Omega_\varepsilon)} + \|\partial_s g\|^2_{L^2(\Omega_\varepsilon)} \le \kappa_3;$$

$$\|\partial_{xx} g\|^2_{L^2(\Omega_\varepsilon)} + \|\partial_{xs} g\|^2_{L^2(\Omega_\varepsilon)} + \|\partial_{ss} g\|^2_{L^2(\Omega_\varepsilon)} \le \frac{\kappa_4}{\varepsilon^{2+0}}.$$

Let us assume that all these functions and their derivatives are continuous on Ω_ε. Taking into account that the square of Ω_ε is equal to ε and the power of ε in the r.h.s. of above inequalities is less then 1, we therefore infer that maximum of absolute value of all these functions and their derivatives tends to ∞, as $\varepsilon \to +0$. It means that the set of initial conditions $\mathbb{E}_2$ increase in $\mathcal{E}_2$ when $\varepsilon \to 0$.

Corollary 1. *There exist the constants ρ_1 and ρ_2, small enough, such that if $f \in H^1_{per}$, $g \in H^2_{per}$ and*

$$\|g\|^2_1 + \|f\|^2_0 \le \rho_1, \tag{35}$$

$(m_0, n_0, E_0) \in \mathcal{E}_2$ and

$$\|m_0\|^2_0 + \lambda^2\|n_0\|^2_1 + 2\lambda^2\|E_0\|^2_2 \le \rho_2, \tag{36}$$

then system (2) with $\varepsilon = 1$ has a unique strong solution, which belongs to $C(\mathbb{R}, \mathcal{E}_2)$.

Since for fixed $\varepsilon = 1$ we can choose the external forces and the initial data small enough, such that conditions (3) and (4) remains true, the proof of this corollary is trivial.

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References

[1] *M. Abounouh and O. Goubet*, Attractor for a damped cubic schrodinger equation on a two-dimensionsl thin domain. — *Dif. Integr. Equations* (2000), v. 13, p. 311–340.

[2] *I.D. Chueshov*, Introduction to the theory of infinite-dimensional dissipative systems. Acta, Kharkov (1999).

[3] *I. Flahaut*, Attractors for the dissipative Zakharov system. — *Nonlinear Anal.* (1991), v. 16, p. 599–633.

[4] *O. Goubet and I. Moise*, Attractors for the dissipative Zakharov system. — *Nonlinear Anal.* (1998), v. 7, p. 823–847.

[5] *J. Hale and G. Raugel*, A damped hyperbolic equation on thin domains. — *Trans. Amer. Math. Soc.* (1992), v. 329, p. 185–219.

[6] *J. Hale and G. Raugel*, Reaction-diffusion equation on thin domains. — *J. Math. Pures Appl.* (1992), v. 71. p. 33–95.

[7] *J.L. Lions and E. Magenes* , Problemes aux Limites Non Homogenes et Applications. Dunod, Paris (1968).

[8] *G. Raugel and G. Sell*, Navier–Stokes equations on thin 3D domains. I: Global attractors and global regularity of solutions. — *J. Amer. Math. Soc.* (1993), v. 6, p. 503–568.

[9] *A.S. Shcherbina*, Gevrey regularity of the global attractor for the dissipative Zakharov system. — *Dynamical Systems* (2003), v. 18, p. 201–225.

[10] *R. Temam*, Infinite-dimensional dynamical systems in mechanics and physics. Springer–Verlag, New York (1988).

[11] *V.E. Zakharov*, Collapse of Langmuir waves. — *Soviet Phys. JETP* (1972), v. 35, p. 908–912.